Solitary Burnup Wave Solution in a Multi-Group Diffusion-Burnup Coupled System

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General Background

Well-known Figures and Facts:

- 1 kg ^{235}U fission reaction produces 23 million kWh.
- Natural uranium possesses only 0.7 % ^{235}U and 99.2% ^{238}U. Fuel enrichment is normally needed
- Conventional light water reactors can only consume 2-5% heavy isotopes (^{235}U, ^{238}U, ^{239}Pu …). Fast breeder reactors about 10% av. burnup. Fuel reprocessing is needed.

Conventional Nuclear Reactors :

- Low fuel utilization and high amount of nuclear waste due to the low burn-up

Burning Wave Fission Reactor: CANDLE

Main features of this concept :

- Long core or continuous fuel loading
- Stable nuclear reaction (constant reactivity)
- High burn-up (up to 50 %) (high utilization of fuel)
- Natural uranium fuel (no need of fuel enrichment)
- Less nuclear waste (no need of fuel reprocessing)

Purpose of this study :

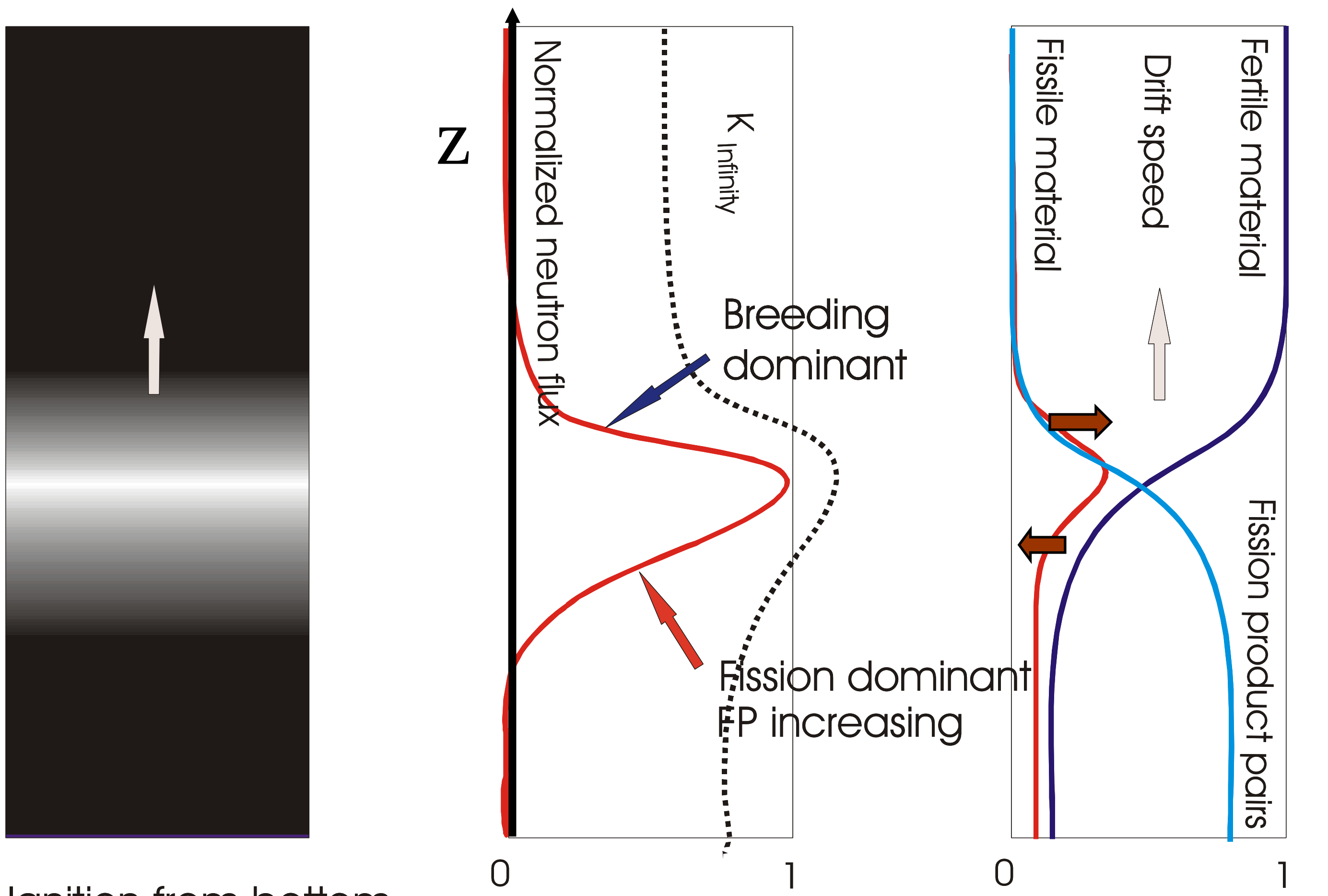
- Setup of simplified multi-group model
- Analytical/numerical solution
- Insight into this phenomenon

Literature:

- **Feoktistov (1989), Teller *et al*. (ICENES'96, Obninsk)**
- **Sekimoto *et al*. (2000, 2001, 2002, 2005)**
- **Van Dam (2000)**
- **Seifritz (2000, 2005)**
- **Chen *et al*. (2005, 2006)**

This paper attempts to apply the analytical method to the more complicated multi-group model

Mechanism of this new kind of reactor



Ignition from bottom

Fresh fuel: fertile fuel

Spent fuel+FPs

Neutronic Model (1)

Two-group diffusion equations (fast group 1; slow group 2):

$$\frac{1}{v_1}\frac{\partial \phi_1}{\partial t} = \nabla\cdot\left(D_1\nabla\phi_1\right) - \Sigma_{a1}\phi_1 - \Sigma_{1\to 2}\phi_1 + \nu_1\Sigma_{f1}\phi_1 + \nu_2\Sigma_{f2}\phi_2,$$

$$\frac{1}{v_2}\frac{\partial \phi_2}{\partial t} = \nabla\cdot\left(D_2\nabla\phi_2\right) - \Sigma_{a2}\phi_2 + \Sigma_{1\to 2}\phi_1,$$

One-group diffusion equations is derived as:

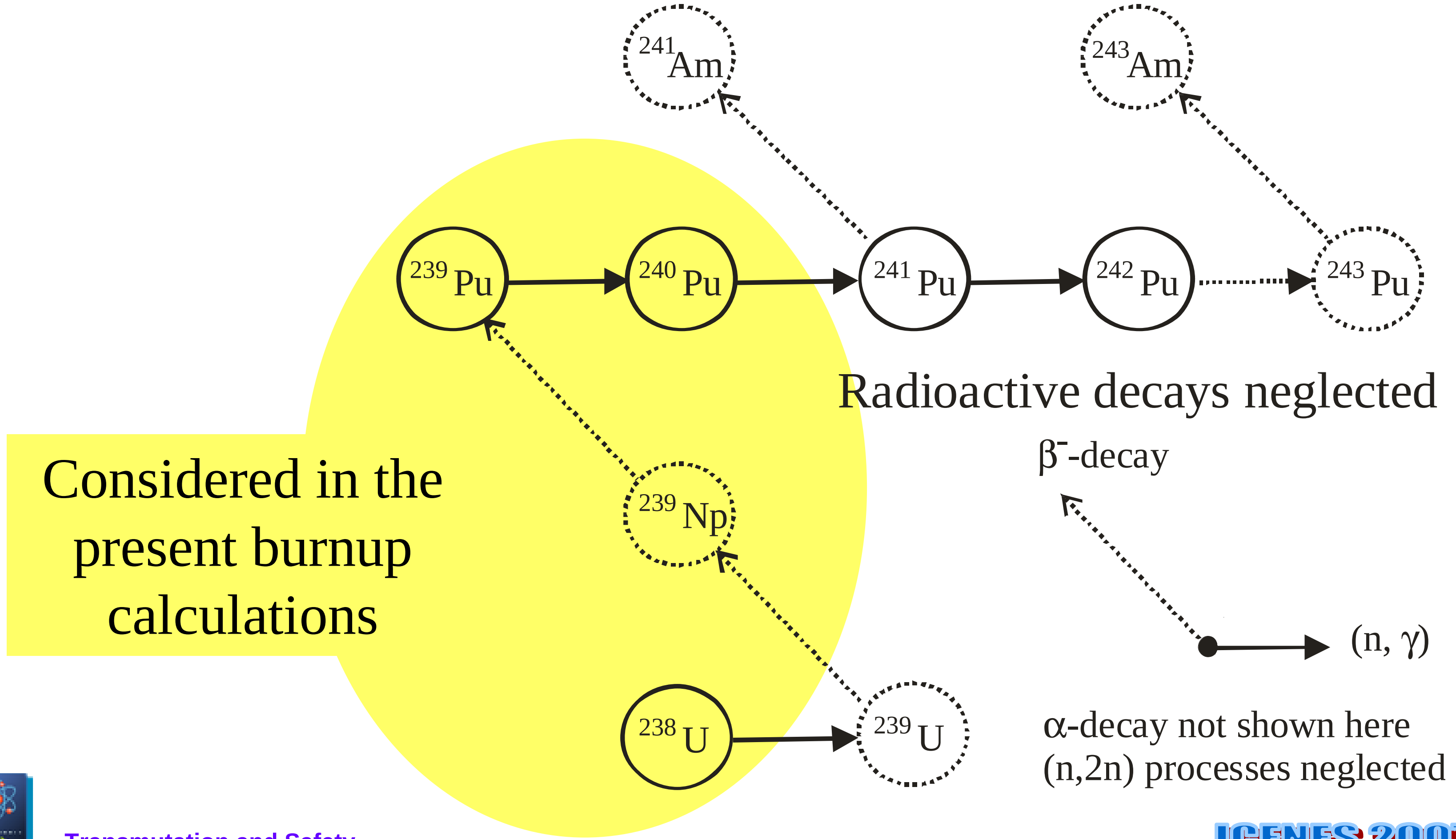
$$\frac{1}{v}\frac{\partial \phi}{\partial t} = \nabla\cdot\left(D\nabla\phi\right) - \Sigma_a\phi + \nu\Sigma_f\phi,$$

where, e.g.

$$D\nabla\phi = D_1\nabla\phi_1 + D_2\nabla\phi_2 \qquad \Sigma\phi = \Sigma_1\phi_1 + \Sigma_2\phi_2 \qquad \phi = \phi_1 + \phi_2$$

Neutronic Model (2)

Simplified Nuclide Chain Scheme



Neutronic Model (3)

Burn-up equations simplified by considering ^{238}U, ^{239}Pu, ^{240}Pu, and burnable and non-burnable fission product pairs (FPPs) and by neglecting radioactive decay processes

$$\frac{\partial N_8}{\partial t} = -N_8 \sigma_{a,8}\phi, \quad \frac{\partial N_9}{\partial t} = -N_9 \sigma_{a,9}\phi + N_8 \sigma_{c,8}\phi, \quad \frac{\partial N_0}{\partial t} = -N_0 \sigma_{a,0}\phi + N_9 \sigma_{c,9}\phi,$$

$$\frac{\partial N_{FPP\_burn}}{\partial t} = -N_{FPP\_burn}\sigma_{a,FPP}\phi + \sum_{i=8,9,0} N_i \sigma_{f,i}\phi, \quad \frac{\partial N_{FPP\_inert}}{\partial t} = N_{FPP\_burn}\sigma_{a,FPP}\phi$$

where all $\sigma\phi$ **stands for** $\sigma_1\phi_1 + \sigma_2\phi_2$

Coupling through macroscopic cross-sections

$$\Sigma_{a,n} = \sum_i N_i \left(\sigma_{a,i}\right)_n, \nu\Sigma_{f,n} = \sum_i N_i \left(\nu_i \sigma_{f,i}\right)_n, \Sigma_{tr,n} = \sum_i N_i \left(\sigma_{tr,i}\right)_n, \Sigma_{1\to 2} = \sum_i N_i \sigma_{1\to 2,i}.$$

Asymptotic Solution

Solutions of Burn-up equations

Atom number densities are functions of *neutron fluence* ψ only

$$N_i = N_i(\psi), \qquad \psi = \int_{t_0}^{t} \phi \, \mathrm{d}t = \psi_1 + \psi_2$$

Asymptotic Formulation in $\zeta = z + ut$

Look for a so-called travelling wave solution with drift speed *u*, *i.e.* a steady solution in a moving coordinate system Oζ in 1-D case

$$\frac{\partial}{\partial\zeta}\left(D\frac{\partial}{\partial\zeta}\phi\right) - \Sigma_a\phi + \frac{\nu\Sigma_f}{k_{eff}}\phi = 0, \qquad \frac{\partial}{\partial\zeta}\left(D_2\frac{\partial}{\partial\zeta}\phi_2\right) - \left(\Sigma_{a2} + \Sigma_{1\to2}\right)\phi_2 + \Sigma_{1\to2}\phi = 0.$$

⇩ ϕ-equation

⇩ ϕ_2-equation

Iterative Solution Method

The analytical solvability of one-group equation (Chen *et al*. ICENES 2005), i.e. if $\Sigma_a(\psi)$, $\Sigma_f(\psi)$ and $D(\psi)$ are known, the ϕ-equation can be solved analytically, where $\psi = \int \phi d\zeta / u$

The two-group equations can be solved **iteratively** based on this one group solution:

(1) Select a suitable constant $\alpha^{(0)} = \phi_2 / \phi$, an analytical solution of ϕ-equation and a numerical solution of ϕ_2-equation are obtained

(2) Then $\alpha^{(1)}(\psi) = \phi_2^{(1)}(\psi) / \phi^{(1)}(\psi) \Longrightarrow k_{eff}^{(2)}, \phi^{(2)}(\psi)$ and $\phi_2^{(2)}(\psi)$

(3) $\alpha^{(n)}(\psi) = \phi_2^{(n)}(\psi) / \phi^{(n)}(\psi) \Longrightarrow k_{eff}^{(n+1)}, \phi^{(n+1)}(\psi)$ and $\phi_2^{(n+1)}(\psi)$

If $\alpha^{(n)}$ is sufficently close to $\alpha^{(n-1)}$, we say, a converged solution has been achieved

Example

- Material composition similar to that in a "simplified standard system" (**sodium-cooled fast breeder reactor** with oxide fuel)
- Two-group data have been generated with boundary 111keV.
- Fresh fuel parameters:

$$\sigma_{a,0} = 0.528\,\text{barn} \qquad N_{8,0} + N_{9,0} = 6.32\times10^{21}\,\text{cm}^{-3}$$

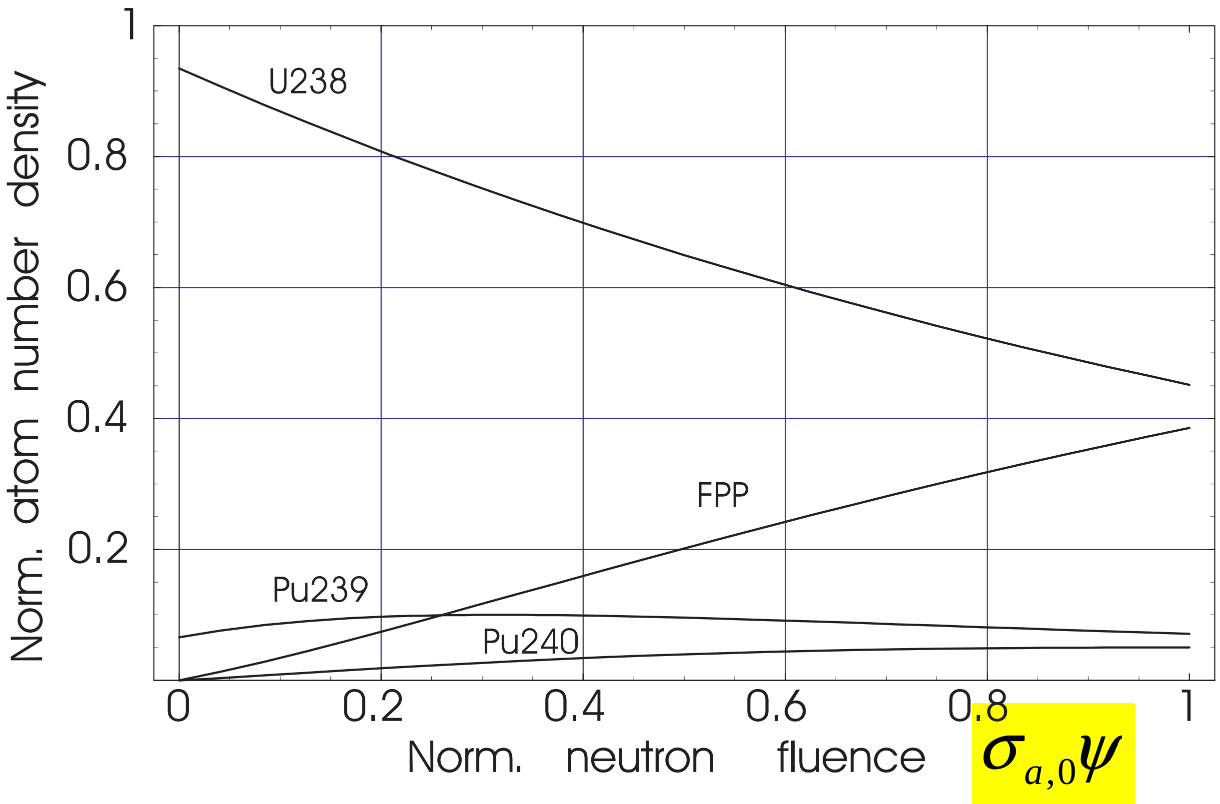
$$D_0 = 1.556\,\text{cm} \qquad N_{9,0}/(N_{8,0} + N_{9,0}) = 0.065663$$

- Maximal neutron flux is chosen as $\phi_{\max} = 3\times10^{15}\,\text{cm}^{-2}\text{s}^{-1}$
- Derived scales

$$l_0 = 21.6\,\text{cm} \qquad u_0 = 3.42\times10^{-8}\,\text{cm/s} \qquad \text{for } Z = \zeta/l_0, \quad U = u/u_0$$

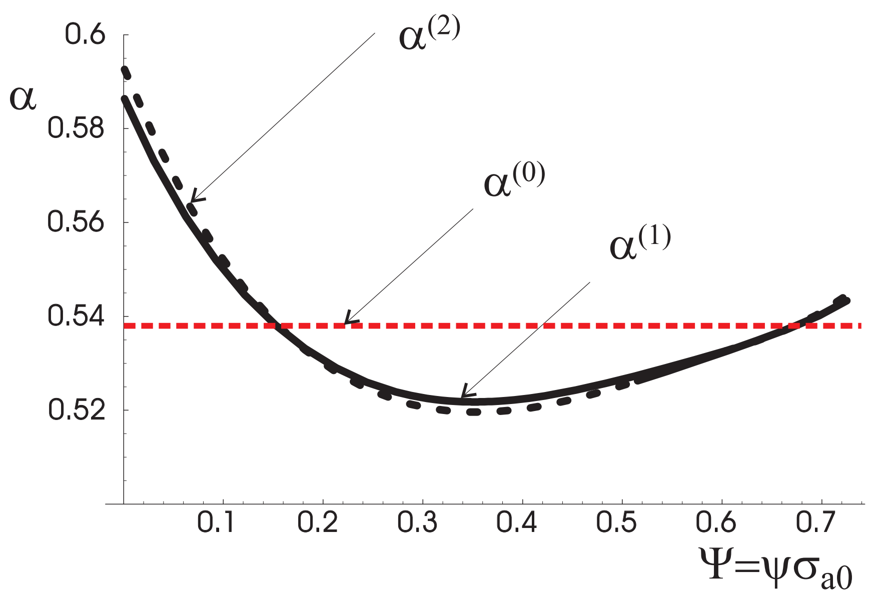
Results (1)

- A typical burn-up solution in the case of constant microscopic cross sections

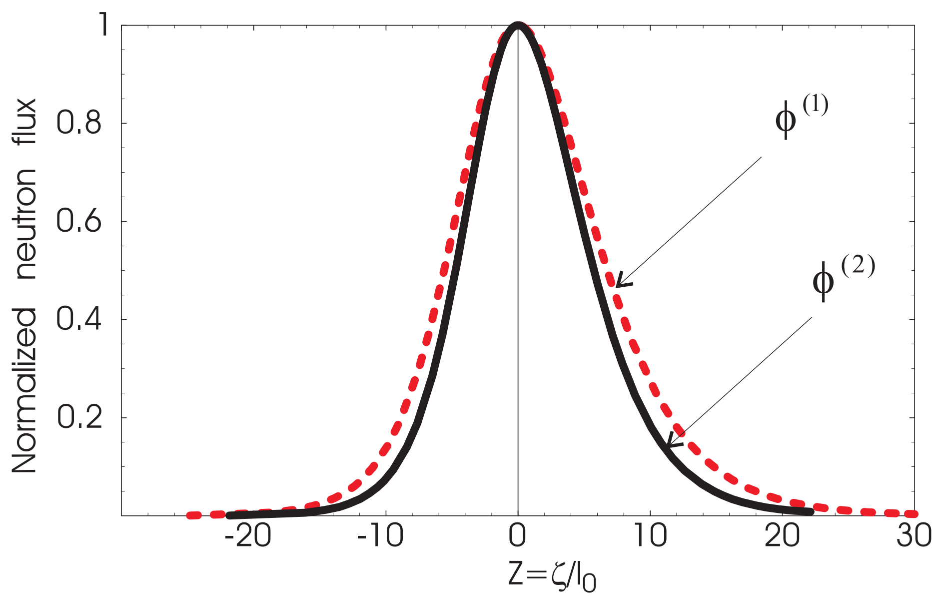


Results (2)

- Comparison of one-group and two-group solutions



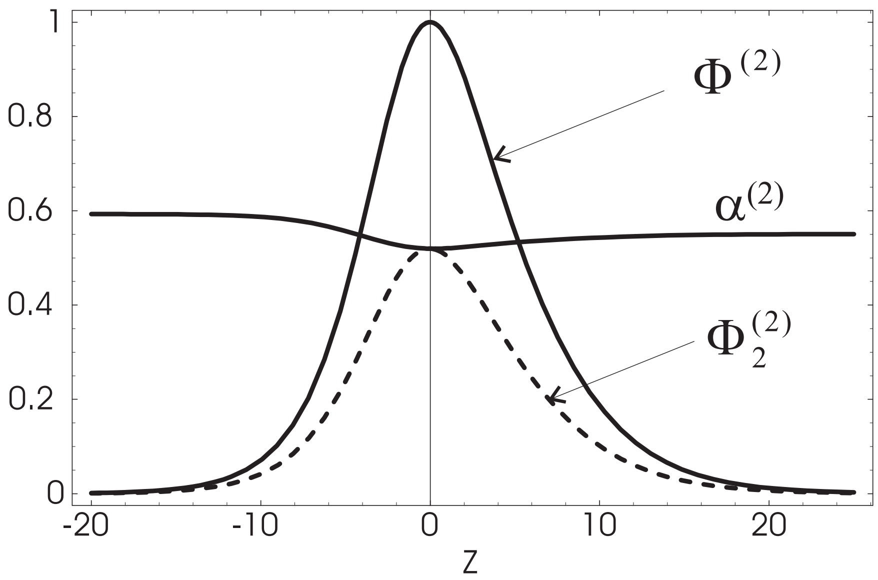
Flux fraction of slow group



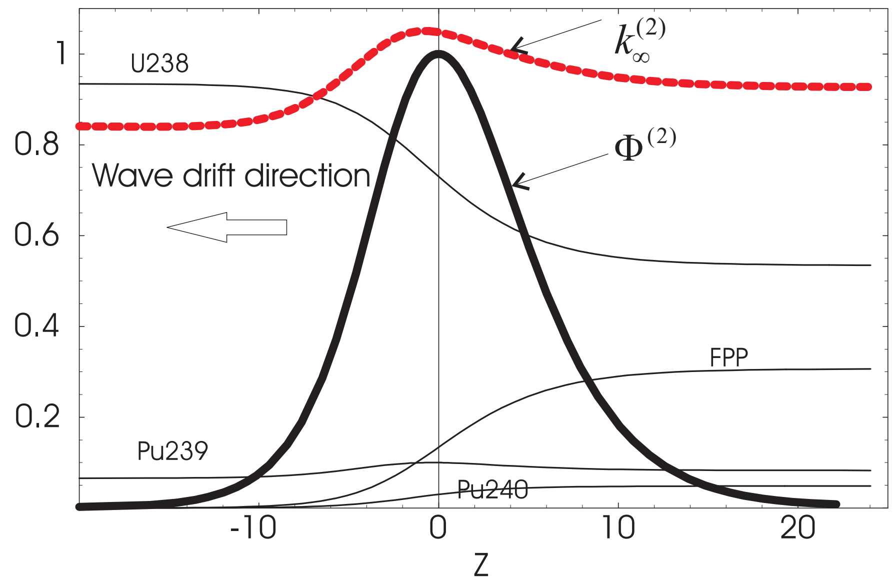
Total neutron flux

Results (3)

- Summary of the two-group solution



Slow neutron flux fraction



2-group solution summary

- The drift speed is 16.7 cm/year and the Z-range of (-20,20) corresponds 8.64 m

Conclusions

- Two-group diffusion equations coupled with simplified burn-up equations were investigated for a CANDLE-type reactor.
- In the 1-D case the model was solved by the iterative method based on the analytical solvability of one-group theory.
- It was shown that the fraction of slow neutron flux changes slightly in the wave solution and consequently the iteration converges very quickly.
- The method can be further developed for a more complicated multi-group model with a large number of energy groups, to obtain a so-called "fundamental burning wave solution".