Study of Inertial Confinement Fusion via the Equation of State

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SOREQ

1. Introduction

2. Why compression

3. Energy gain

4. Bremsstrahlung, degeneracy and the “Clean Fusion” problem

5. Conclusions

Ideal gas in a gravitational field

$$\text{internal energy} = E = \frac{PV}{\gamma-1} = \frac{Nk_BT}{\gamma-1}; \quad \gamma = \frac{C_P}{C_V}$$

$$\text{kinetic energy} = E_{kin} = (3/2)Nk_BT = (3/2)(\gamma-1)E$$

$$\text{virial theorem} \Rightarrow -\sum_{i,j} G\frac{m_i m_j}{\left|\vec{r}_i - \vec{r}_j\right|} = E_{pot} = -2E_{kin} = 3(1-\gamma)E$$

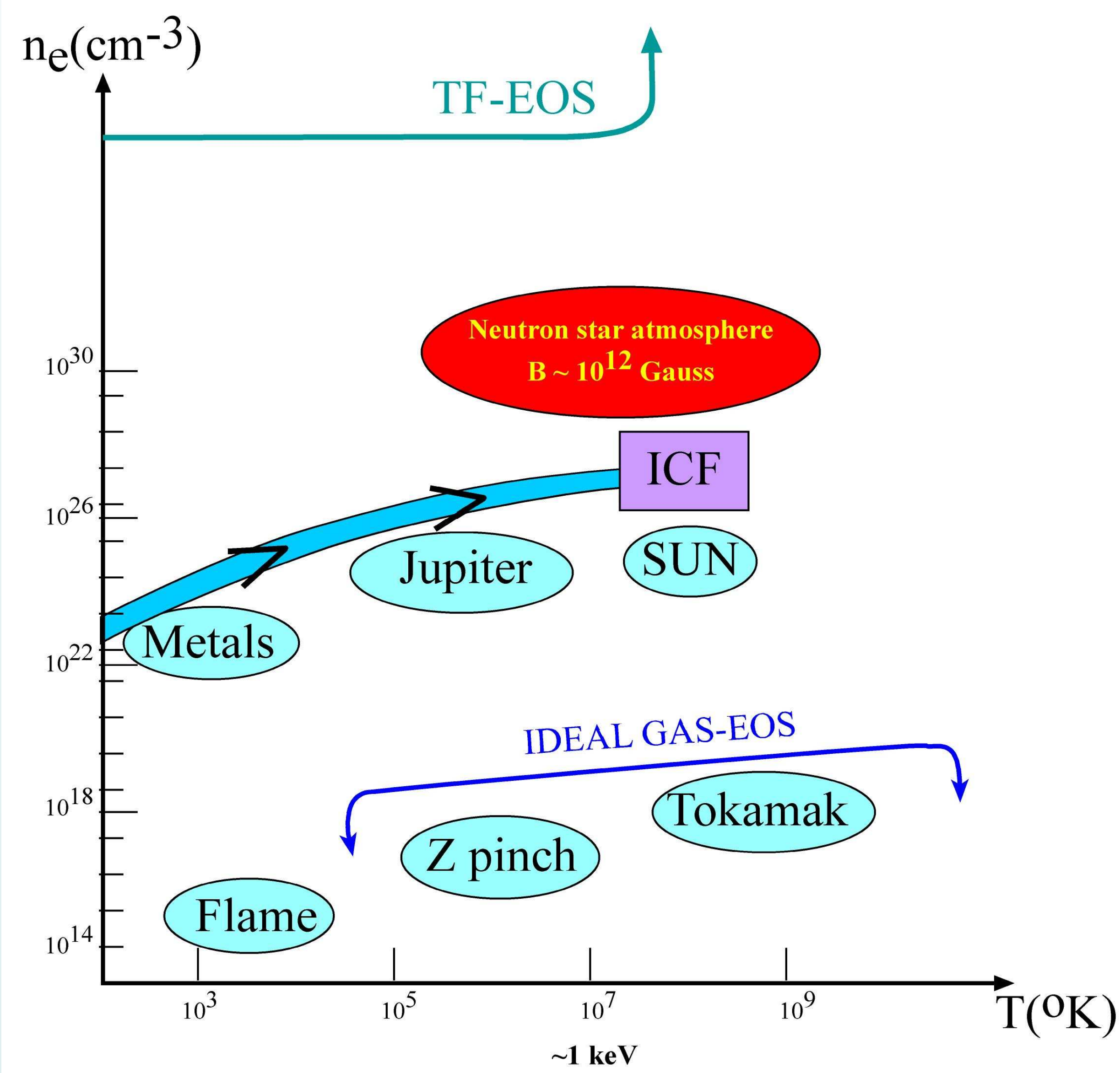
$$E_{total} = E + E_{pot} = \frac{(\gamma-4/3)}{(\gamma-1)}E_{pot}$$

$$E_{total} = \frac{(\gamma - 4/3)}{(\gamma - 1)} E_{pot}$$

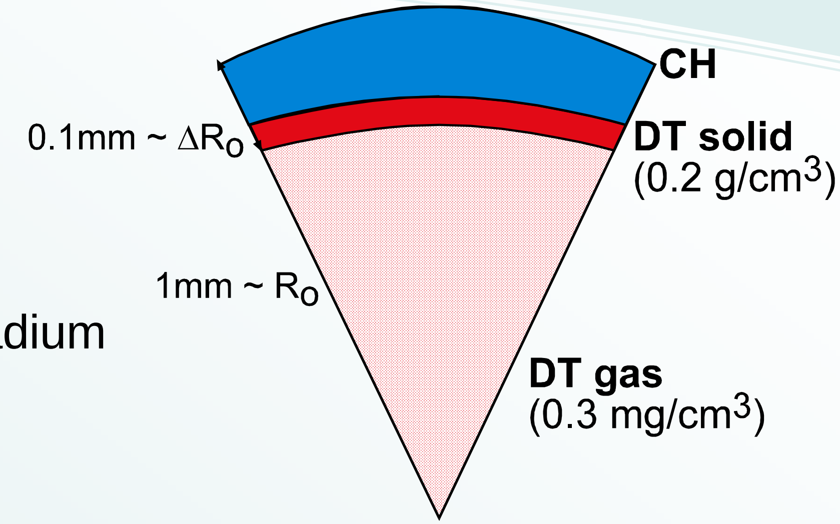
$$\gamma > 4/3 \Rightarrow E_{total} < 0 \Rightarrow \text{star stable}$$

$$\text{virial theorem} \Rightarrow G\frac{M}{R} \sim 2E_{kin} \sim T$$

- **As the star contracts its internal energy increases and it gets hotter. At the same time it radiates energy!**
- **This is due to the fact that loss in potential energy is greater than the increase in internal energy and the differences is radiated away.**
- **For example, for γ= 5/3: only half of the decrease in the potential energy is used in increasing its internal energy (i.e. temperature)**



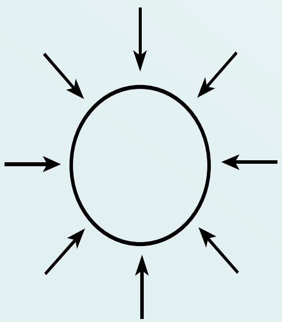
$n_e(cm^{-3})$
TF-EOS
Neutron star atmosphere
B ~ 10^{12} Gauss
ICF
Jupiter
SUN
Metals
IDEAL GAS-EOS
Tokamak
Z pinch
Flame
10^{30}
10^{26}
10^{22}
10^{18}
10^{14}
10^3
10^5
10^7
10^9
$T(^oK)$
~1 keV



LASER SYSTEM ~ football stadium

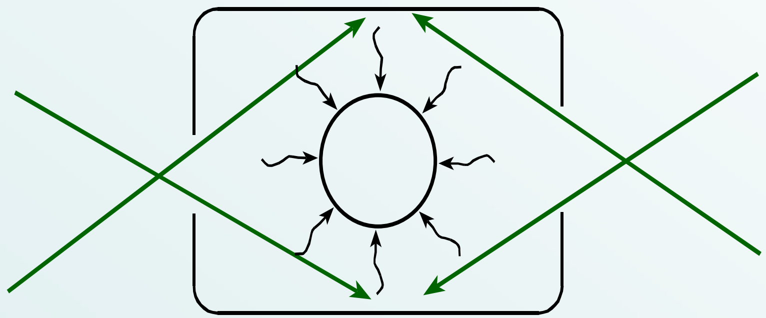
DIRECT DRIVE

Laser beams shine on a pellet directly



INDIRECT DRIVE

Laser beams shine on a miniature oven(=hohlraum) which heats up the pellet by x-rays



x-t is space-time,

ρ = density, u = flow velocity,

P = pressure, E = internal energy

3 hydrodynamic equations,

4 variables (ρ, u, P, E),

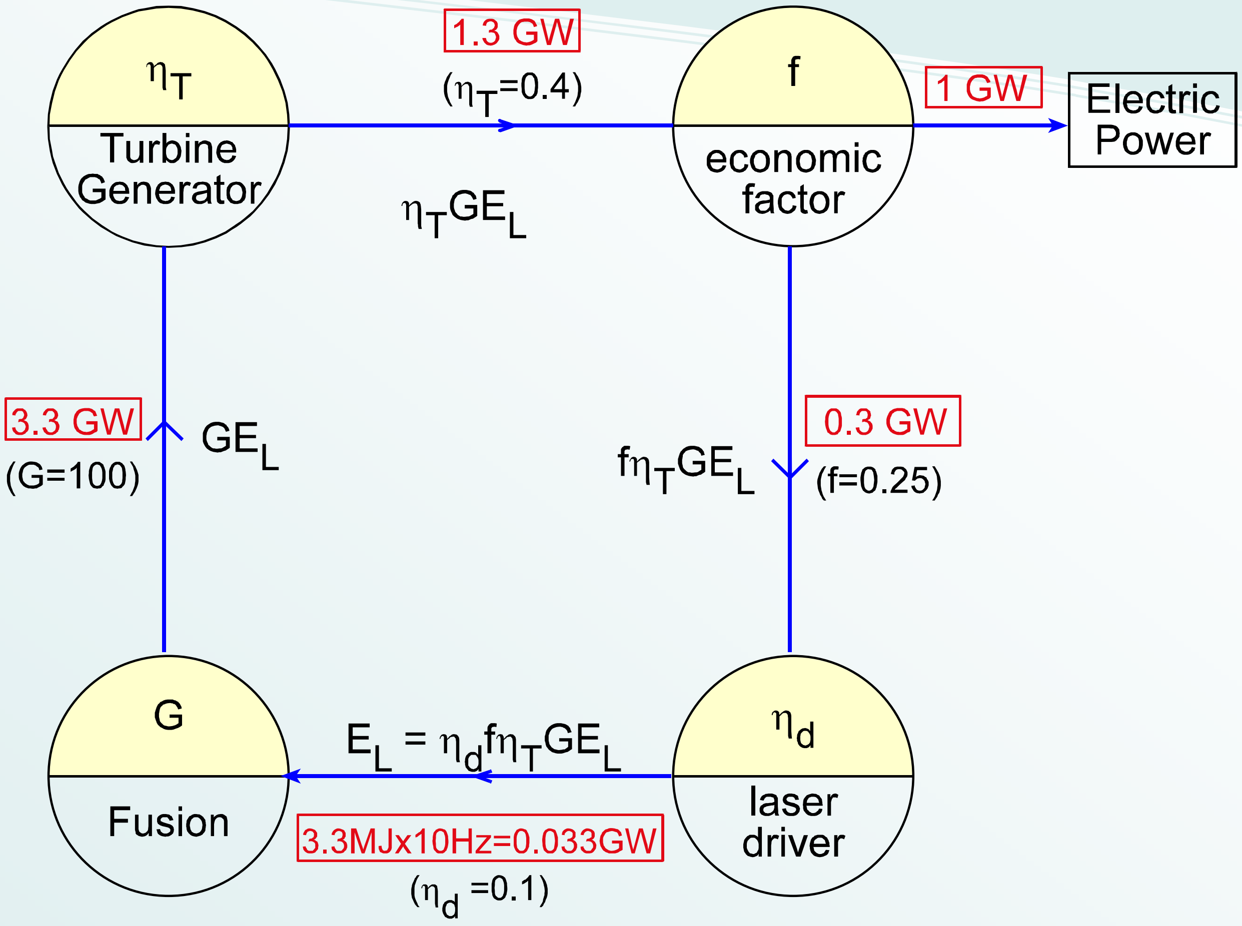
The fourth equation is EOS!

$$\text{Mass:} \qquad \frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial x}(\rho u)$$

$$\text{Momentum} \qquad \frac{\partial}{\partial t}(\rho u) = -\frac{\partial}{\partial x}\left(P + \rho u^2\right)$$

$$\text{Energy:} \ \frac{\partial}{\partial t}\left(\rho E + \frac{1}{2}\rho u^2\right) = -\frac{\partial}{\partial x}\left(\rho E u + P u + \frac{1}{2}\rho u^3\right)$$

$$\text{EOS:} \qquad E = E(P,\rho)$$



$\Rightarrow$ $G\eta_d \, \eta_T \, f=1$

$G \gtrsim 100$

Example: $\eta_d = 0.1$, $\eta_T = 0.4$, $f=0.25 \Rightarrow G = 100$

2. WHY COMPRESSION?

η_H = (hydrodynamic) efficiency from laser to plasma

η_A = laser absorption by pellet

$$\eta_H \eta_A E_L = (3 n_C k_B T)\left(\frac{4}{3}\pi R_C^3\right)$$

$$E_L = \left(\frac{1}{\eta_H \eta_A}\right)\left(\frac{4\pi}{2.5 m_p \rho_0^2}\right)\left(\frac{n_0}{n_c}\right)^2 (\rho_C R_C)^3 k_B T$$

using:

$\rho_0 = 0.2[g/cm^3]$, $\rho_C R_C = 3[g/cm^2]$ $\{$for $\phi \approx 30\%\}$

$k_B T = 4keV$ $\{$for E(fusion) > E(bremsstrahlung)$\}$

$$\Rightarrow E_L[MJ] \approx \left(\frac{1.3 \cdot 10^6}{\eta_H \eta_A}\right)\left(\frac{\rho_0}{\rho_C}\right)^2$$

$$\text{example: } \eta_H = 0.1,\ \eta_A = 0.8,$$

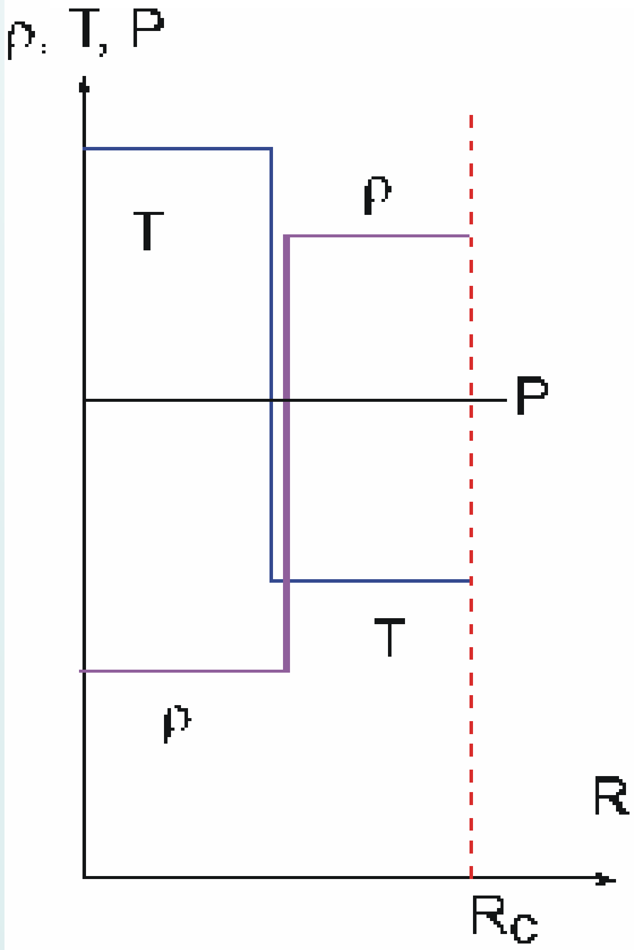
$$\Rightarrow E_L[\text{MJ}] \approx 1.6 \cdot 10^7 \left(\frac{\rho_0}{\rho_C} \right)^2$$

$$\frac{\rho_C}{\rho_0} = 3000 \ \left\{ \text{i.e. } \rho_C = 600[\text{g/cm}^3] \right\}$$

$$\Rightarrow E_L[\text{MJ}] \approx 1.75$$

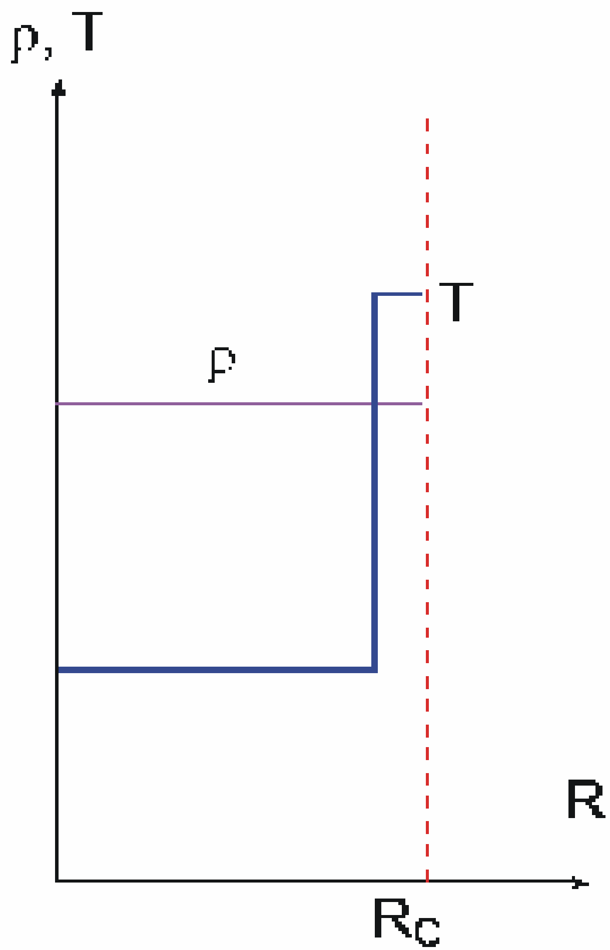
Spark Ignition

(isobaric: P=const)

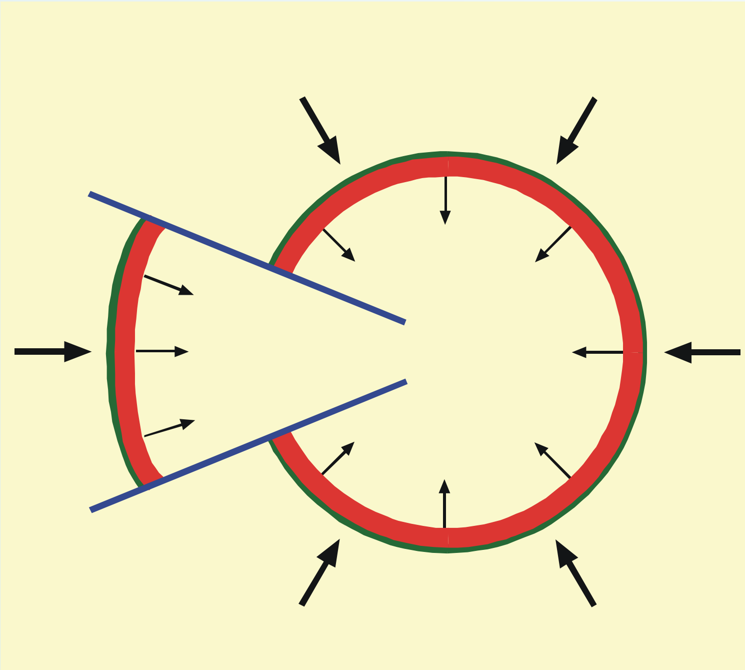


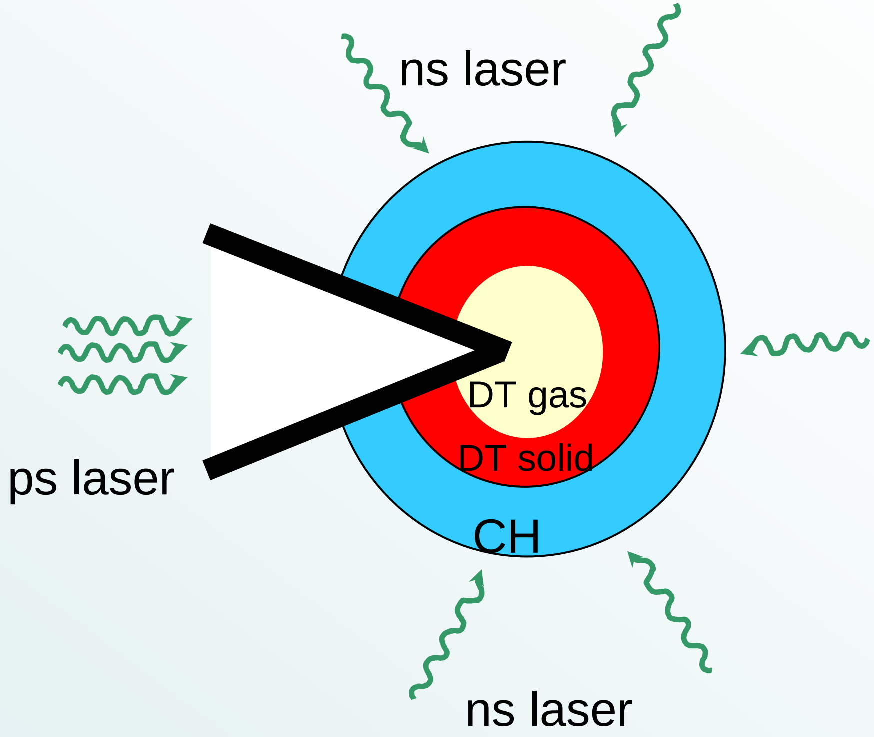
Fast Ignition

(isochoric: ρ=const)



3. ENERGY GAIN
FAST IGNITION:
Impact and Laser





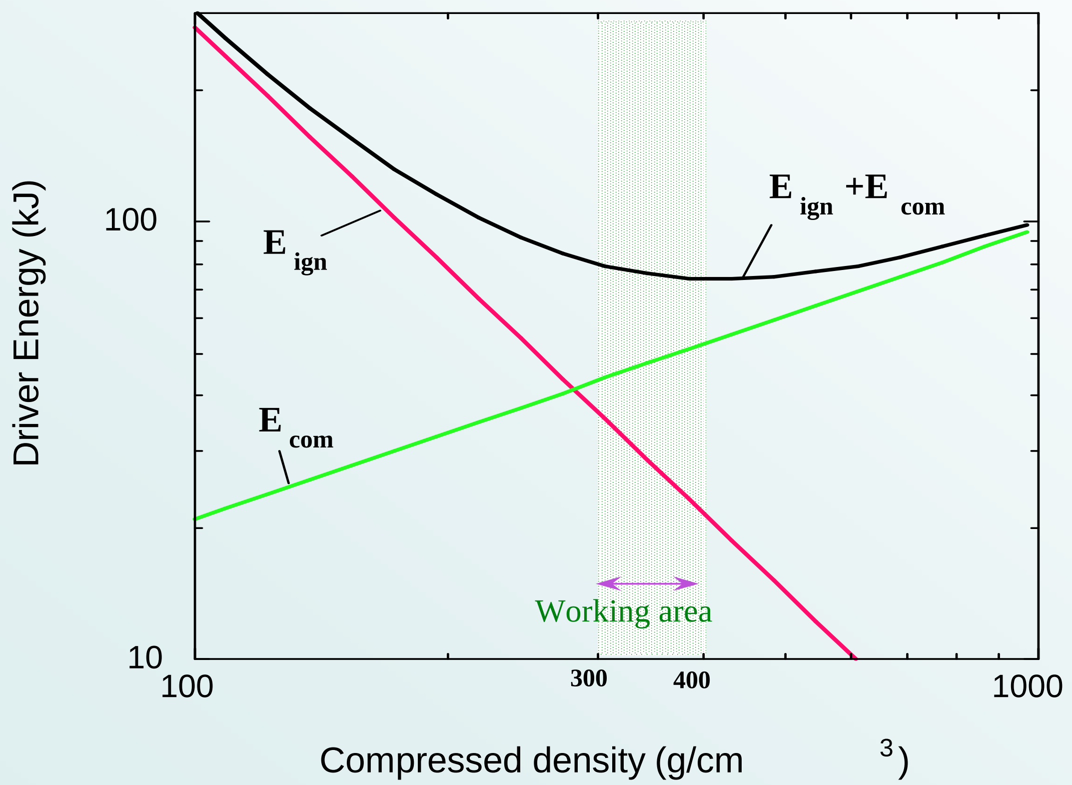
Gain in Impact Fast Ignition

$$E_{ign} \approx 15[kJ]\left(\frac{1}{\eta_S}\right)\left(\frac{100g/cm^3}{\rho_S}\right)^2$$

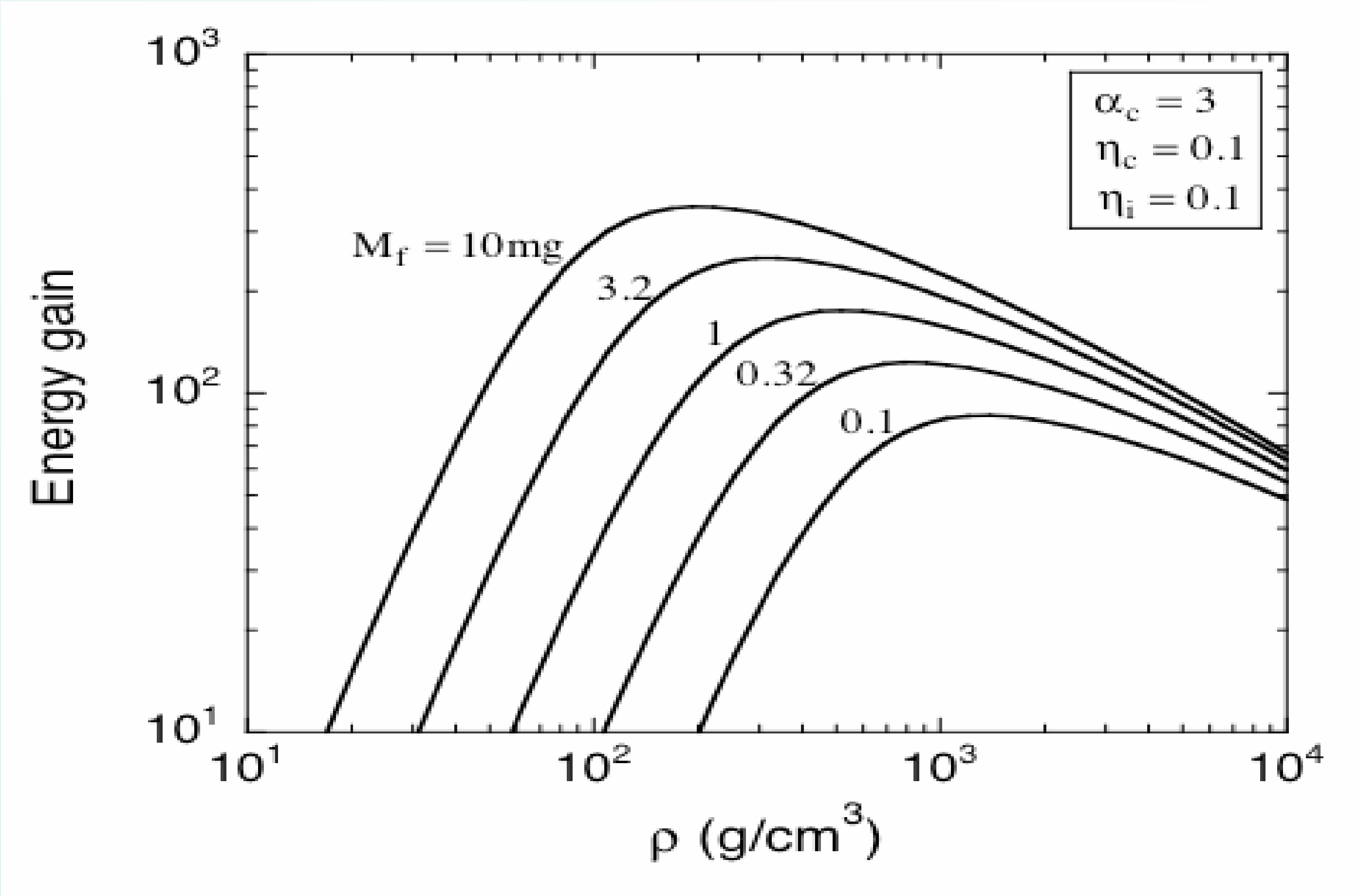
$$E_{com} \approx 110[kJ]\left(\frac{1}{\eta_C}\right)\alpha_C\left(\frac{\rho_C}{\rho_0}\right)^{2/3}\left(\frac{M_C}{g}\right)$$

$$\text{Total Driver Energy} \equiv E_d = E_{com} + E_{ign}$$

$$\text{GAIN} \equiv G = \frac{E_{fusion}}{E_d} \approx \frac{\phi q_{DT} M_C}{E_d}$$



Driver Energy (kJ)
100
10
E ign
E com
E ign +E com
Working area
100
300
400
1000
Compressed density (g/cm 3)



$\alpha_c = 3$
$\eta_c = 0.1$
$\eta_i = 0.1$
$M_f = 10mg$
3.2
1
0.32
0.1
Energy gain
10^3
10^2
10^1
10^1
10^2
10^3
10^4
ρ (g/cm^3)

4. Bremsstrahlung, degeneracy and the p^{11}B problem

$$\left.\frac{W_{deg}}{W_{class}}\right\}_{bremss} = \left(\frac{\sqrt{\pi}}{2F_{1/2}(\eta)}\right)\left\{F_1(\eta) - \frac{1}{2}\left[\ln\left(e^{\eta}+1\right)\right]^2\right\}$$

$$F_k = \int_0^{\infty} \frac{x^k dx}{e^{x-\eta}+1}$$

$$\eta \equiv \frac{T}{\varepsilon_F} << 1 \Rightarrow \left.\frac{W_{deg}}{W_{class}}\right\}_{bremss} = \frac{\pi^2\sqrt{\pi}}{8}\left(\frac{T}{\varepsilon_F}\right)^{3/2}$$

For the DT case we need a density ~ 5×10^4 g/cm^3

for $p^{11}B$ the density has to as large as ~ 2×10^7 g/cm^3

$$\varepsilon_F = \left(3\pi^2\right)^{2/3}\left(\frac{\hbar^2}{2m_e}\right){n_e}^{2/3} \Rightarrow \varepsilon_F = \varepsilon_{F0}\left(\frac{Z}{Z_0}\right)^{2/3}\left(\frac{\rho}{\rho_0}\right)^{2/3}$$

$$\text{degeneracy case: } \varepsilon_F >> T \Rightarrow \varepsilon_F \approx 5T$$

$$\text{for DT:} \quad T \sim 2\text{keV}$$

$$\text{for } p^{11}B\text{:} \quad T \sim 100\text{keV}$$

$$\mathrm{D} + \mathrm{T} \rightarrow \mathrm{n} + \alpha + 17.6\,\mathrm{MeV}$$

$$\text{(a)}\quad G_{max}\left(\mathrm{DT2e}\right)_{deg} = \frac{17{,}600\,\mathrm{keV}}{2\varepsilon_F + 2\times\left(3T/2\right)} \approx \frac{17{,}600\,\mathrm{keV}}{2.6\varepsilon_F} \approx 677$$

$$\text{(b)}\quad G_{max}\left(\mathrm{DT2e}\right)_{clas} = \frac{17{,}600\,\mathrm{keV}}{4\times\left(3T/2\right)} \approx \frac{17{,}600\,\mathrm{keV}}{6\times 4\,\mathrm{keV}} \approx 733$$

Theoretical Maximum Gain for

$$p + {}^{11}B \rightarrow 3\alpha + 8.7\,MeV$$

$$\text{(a)}\quad G_{max}\left(pB^{11}5e\right)_{deg} = \frac{8,700\,keV}{5\varepsilon_F + 2\times\left(3T/2\right)} \approx \frac{8,700\,keV}{5.6\varepsilon_F} \approx \frac{8700}{2800} \approx 3.1$$

$$\text{(b)}\quad G_{max}\left(pB^{11}5e\right)_{clas} = \frac{8,700\,keV}{7\times\left(3T/2\right)} \approx \frac{8,700\,keV}{10.5\times 100\,keV} \approx 8.2$$

$$\text{(c)}\quad G_{max}\left(pB^{11}5e\right)_{ion\ only} = \frac{8,700\,keV}{2\times\left(3T/2\right)} \approx \frac{8,700\,keV}{300\,keV} \approx 30$$

CONCLUSION 1

NO CLEAN FUSION!!

$$p + {}^{11}B \rightarrow 3\alpha + 8.7\,\mathrm{MeV}$$

CONCLUSION 2:

If DT fusion successful (economically)

then we have to master **tritium** and **neutrons**,

even for DD fusion or DHe3 fusion

$$\mathrm{D+D} \rightarrow \begin{cases} \mathrm{p+T+4.1\ MeV} \\ \mathrm{n+He3+3.2\ MeV} \end{cases}$$

$$\mathrm{D + He3 \rightarrow p + \alpha + 18.3\ MeV}$$

Appendix

SPARK IGNITION

ξ = the fraction of the heated fuel ≈ 0.14

the rest is heated by α particles

For $\rho R \geq 0.3$ [g/cm^2] α is absorbed!

$$\Rightarrow G = \frac{\eta_H \eta_A \phi q_{DT}}{\xi q_T} \approx 100$$

PROBLEMS: INSTABILITIES (R-T, etc), MIXING

$\Rightarrow$ NO GAIN!

A SOLUTION: FAST IGNITION

THE HOT SPOT

$T_S \approx 5$ keV (fusion energy > bremsstrahlung losses)

$H_S \equiv \rho_S R_S \approx 0.4$ [g/cm^2] (α is absorbed for 0.3 [g/cm^2])

E_{ign} = laser energy directed into "hot spot"

η_S = efficiency from laser to thermal energy

v_i = implosion velocity

$m_i = 2.5 m_p$ (i.e. DT fusion)

M_s = total mass of ignitor

kinetic energy = thermal energy

THE HOT SPOT

$$T_S \approx 5 \text{ keV } \left(\text{fusion energy} > \text{bremsstrahlung losses}\right)$$

$$H_S \equiv \rho_S R_S \approx 0.4 \ [\text{g/cm}^2] \ \left(\alpha \text{ is absorbed for } 0.3 \ [\text{g/cm}^2]\right)$$

$$E_{ign} = \text{laser energy directed into "hot spot"}$$

$$\eta_S = \text{efficiency from laser to thermal energy}$$

$$v_i = \text{implosion velocity}$$

$$m_i = 2.5 m_p \ \text{(i.e. DT fusion)}$$

$$M_s = \text{total mass of ignitor}$$

$$\text{kinetic energy} = \text{thermal energy}$$

$$\Rightarrow \frac{1}{2} m_i v_i^2 = \frac{3}{2}(k_B T_S) \times 2$$

$$\Rightarrow v_i \approx 1.1 \cdot 10^8 \text{ [cm/s]}$$

$$M_S = \frac{4}{3}\pi R_S{}^3 \rho_S = \frac{4\pi H_S{}^3}{3\rho_S{}^2} = 2.7 \cdot 10^{-5}[\text{g}]\left(\frac{100\text{g}/\text{cm}^2}{\rho_S}\right)^2$$

Energy conservation: $\eta_S E_{ign} = 3(k_B T_S)\left(\frac{T_S}{m_i}\right)$

$$\Rightarrow E_{ign} = \frac{8\pi H_S{}^3 (k_B T_S)}{5 m_p \eta_S \rho_S{}^2} = 15[\text{kJ}]\left(\frac{1}{\eta_S}\right)\left(\frac{100\text{g}/\text{cm}^2}{\rho_S}\right)^2$$

THE MAIN FUEL

$P_C \equiv$ fuel (DT) pressure $= n_i T_i + n_e T_e = 2 n_e T_C \quad (T_i = T_e \equiv T_C)$

$n_e =$ electron density,

main fuel compressed density $\equiv \rho_C = (2.5 \cdot m_p) n_e$

$P_{deg} \equiv$ electron degenerate pressure $= \frac{2}{5} n_e \varepsilon_F$

$$\varepsilon_F = \text{Fermi Energy} = \frac{(3\pi^2)^{2/3} h^2 n_e^{2/3}}{8\pi^2 m_e} \approx 2.25 \cdot 10^{-11} \rho_C^{2/3} \text{ [cgs, for DT]}$$

$$\alpha_C \equiv \frac{P_C}{P_{deg}} = \frac{5 T_C}{\varepsilon_F} \quad [\approx 3 \text{ is a reasonable number}]$$

[α_C is expected to be as small as possible (isentropic compression)]

E_{com} = laser energy directed into the main fuel

η_C = efficiency from laser to thermal energy

$$M_C = \frac{4\pi\rho_C R_C^{\ 3}}{3} = \frac{4\pi H_C^{\ 3}}{3\rho_C^{\ 2}}, \qquad H_C \equiv \rho_C R_C$$

ENERGY CONSERVATION:

$$\eta_C E_{com} = 3\left(\frac{M_C}{2.5 \cdot m_p}\right)\left(k_B T_C\right) \approx 3.3 \cdot 10^{12} \alpha_C \rho_C^{\ 2/3} M_C$$

GAIN

$$\text{Total Driver Energy} \equiv E_d = E_{com} + E_{ign}$$

$$\text{The burn fraction} \equiv \phi \approx \frac{H_C}{H_C + H_0}$$

$$H_C = \left(\frac{3\rho_C{}^2 M_C}{4\pi}\right)^{1/3}, \quad H_0 = 7\ [g/cm^2]$$

burn fraction from hot spot is neglected ($H_S \approx 0.4\ [g/cm^2]$; $\phi_S \approx 0.05$)

$$q_{DT} = (17.6\text{MeV}) \cdot \left(\frac{6.02 \cdot 10^{23}}{5[g]}\right) = 3.39 \cdot 10^{11}\ [J/g]$$

$$\text{GAIN} \equiv G = \frac{E_{fusion}}{E_d} = \frac{\phi q_{DT} M_C}{E_d} + \frac{\phi_S q_{DT} M_S}{E_d} \approx \frac{\phi q_{DT} M_C}{E_d}$$

$$\Rightarrow G = G(\alpha_C, \rho_C, \rho_S, \eta_C, \eta_S, E_d); \qquad T_S = 5[keV], H_S = 0.4[g/cm^2]$$

$$G = \frac{10^6 \phi \eta_C}{\alpha_C \rho_C^{2/3}} \left(1 - \frac{1.5 \cdot 10^{15}}{\eta_S \rho_S^2 E_d}\right) \quad [cgs]$$

$$M_C[g] = \frac{\eta_C E_d}{3.3 \cdot 10^{12} \alpha_C \rho_C^{2/3}} \left(1 - \frac{1.5 \cdot 10^{15}}{\eta_S \rho_S^2 E_d}\right)$$

$$\text{example}: \alpha_C = 3, \eta_C = \eta_S = 0.1, \rho_C = 300[g/cm^3], \rho_S = 200[g/cm^3]$$

$$E_d = 100[kJ] = 10^{12}[erg]$$

$$\Rightarrow M_C = 4.6 \cdot 10^{-4}[g], \qquad G \approx 120$$