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Tunneling Effect Enhanced by Lattice Screening as Main Cold Fusion Mechanism: A Brief Theoretical Overview

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Introduction

- In this work are illustrated the main features of tunneling traveling between two deuterons within a lattice.

- Considering the screening effect due lattice electrons we compare the d-d fusion rate evaluated from different authors assuming different screening efficiency and different d-d potentials.

- Then, we propose a effective potential which describes very well the attractive contribute due to plasmon exchange between two deuterons and by means of it we will compute the d-d fusion rates for different energy values.

- Finally the good agreement between theoretical and experimental results proves the reality of cold fusion phenomena and the reliability of our model.

Established the fusion process in terms of penetration of a particle of energy E in a region classically forbidden whose potential is V, the fusion reaction rate Λ (sec^{-1}) will be determined, according to quantum mechanics, from the following expression:

$$\Lambda = A\left|\psi\left(r_o\right)\right|^2$$

Here A is the nuclear reaction constant obtained from measured cross sections, the probability $\left|\psi\left(r_0\right)\right|^2$ is the square modulus of the inter-particles wave-function, and r_0 is the point of forbidden zone.

Finally it is demonstrated that for a Coulomb potential:

$$\left|\psi(r_0)\right|^2 = \left|\frac{k(r_e)}{k(r_0)}\right| \exp\left\{-\frac{2}{\hbar}\int_{r_0}^{r'} \sqrt{2\mu\left(V-E\right)}\,dr\right\}$$

where

$$k(r) = \frac{\sqrt{2\mu\left(V(r)-E\right)}}{\hbar}$$

and *r'* is the classical turning point.

Here μ is the mass of particle incoming, r_0 is a point within forbidden region, E is the energy of particle, and $k(r_e)$ is the wave number of the zero point oscillation:

$$\frac{\hbar^2 k^2(r_e)}{2\mu} = E$$

Of course the pre-factor of exponential is about 1 and the exponential term is know as Gamow-amplitude. More exactly we define as Gamow-factor:

$$\eta_G(r_0) = \exp\left\{-\frac{1}{\hbar}\int_r^{r_0}\sqrt{2\mu(V-E)}\,dr\right\}$$

The fusion probability between two nuclei of D_2 molecular.

In this case we must consider that the average distance between two deuterons, within the D_2- molecule, is

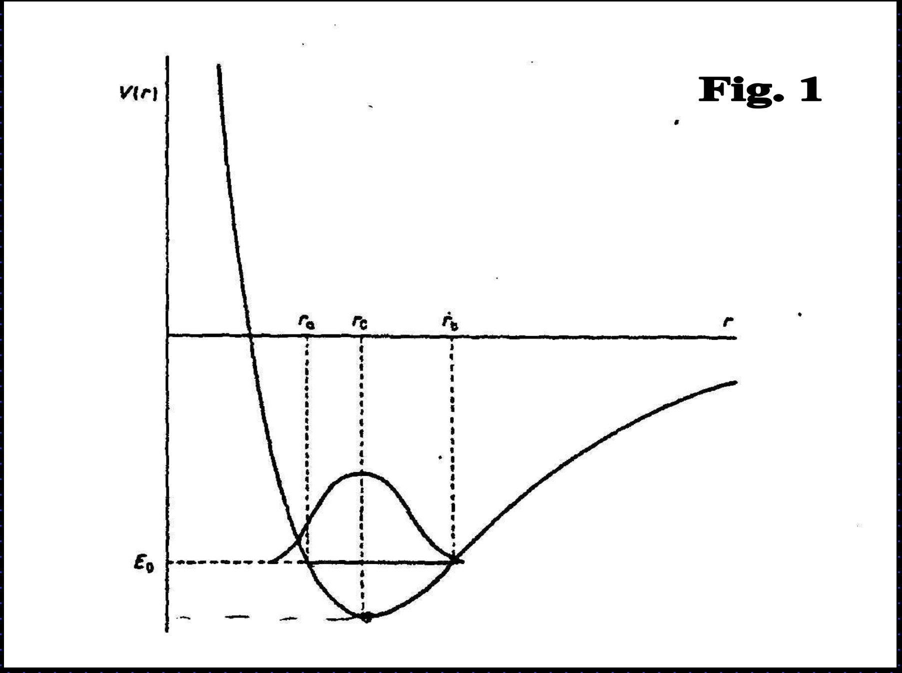
$r_0 \approx 0.74 \overset{0}{A}$ while the distance at which the nuclear force takes place is $r \approx 20\ F$. Then if we put $V=\alpha/r$, i.e. the coulomb potential, and with μ label the reduced mass, we'll compute $k \approx 1$. Finally for $E \approx 0$ we can evaluate:

$$\eta_G(r) = \exp - \left\{ \sqrt{8\alpha\mu} \left(r_0^{1/2} - r^{1/2} \right) \right\} \approx 10^{-60}$$

Using $A=10^{22}\ sec^{-1}$, we obtain $L=10^{-98}\ sec^{-1}$.
It means that no detectable process can take place!

Deuterons Tunneling as probably cold fusion mechanism

In a hydrogen molecular the potential has the trend shown in this figure:



On this case the d-d fusion processes involved are:

1) $d + d \rightarrow {}^{3}He + n$

2) $d + d \rightarrow t + p$

3) $d + d \rightarrow {}^{4}He + \gamma$

Molecular potential energy curve and ground-state vibrational wave-function for the relative motion of two nuclei. The points r_a*,*r_b *and* r_0 *are the classical turning points and the equilibrium inter-nuclear separation, respectively.*

Here a Morse potential is used in the interval that included the inner turning point r_a and continues on toward *r=0*, near which it is connected with the repulsive Coulomb potential *1/r*. The expression is:

$$V(r) = D\left[e^{-2\gamma(r-r_0)} - 2e^{-\gamma(r-r_0)}\right]$$

D is the depth of the potential well that is roughly equal to the dissociation energy and *y* is related to the anharmonicity constant which is a measure of the curvature of the Morse potential well.

Since the vibrational levels of Morse potential can be written in this way

$$E_\nu = -D\left[1 - \frac{\gamma\hbar}{\sqrt{2\mu D}}\left(\nu + \tfrac{1}{2}\right)\right]^2$$

C. De W Van Siclen and S.E. Jones computed
(in units e^2/a_0 and a_0=*Bohr radius*):

D=0.1743

y=1.04

r_0=*1.4*

The molecular wave-function has been evaluated following the method proposed by Langer which now we will briefly illustrate.

The radial part of Scrödinger equation is:

$$\left[\frac{d^2}{dr^2} + Q_0^2(r)\right]\chi(r) = 0$$

where

$$Q_0^2(r) = \frac{2\mu}{\hbar^2}\left[E - V(r)\right] - \frac{J(J+1)}{r^2}$$

and

$$\chi(r) = r\,\psi(r)$$

The ground *v=J=0* molecular wave-function in the interaction region is thus found to be:

$$\psi(r) = \frac{1}{4}\left(\frac{\alpha^2}{\pi^3}\right)^{1/4}\eta$$

with
$$\alpha = \frac{\mu\omega}{\hbar}$$

$$\eta = \exp\left\{-\frac{1}{2}\left[\int_r^{r_0}\left(2|Q(r)| - \frac{1}{r}\right)dr + \ln\frac{r_a}{2}\right]\right\}$$

and
$$Q^2(r) = \frac{2\mu}{\hbar^2}[E - V(r)] - \frac{(J+1/2)^2}{r^2}$$

where V(r) is the coulomb potential for r <ρ and the Morse potential for r >ρ being ρ the point at which the Morse potential is connected to the purely Coulomb potential.

Using these results Van Siclen ad Jones demonstrated the possibility of scaling down of repulsive effect between two deuterons. Moreover they showed that the average fusion rate was much more sensitive to the choice of *ρ* than *r* (see the table):

Table 1

r	ρ	Λ (sec-1)
0	0.4	$3.8\,x10^{-70}$
0	0.5	$1.3\,x10^{-74}$
10^{-3}	0.5	$1.3\,x10^{-74}$
10^{-3}	0.5	$2.3\,x10^{-74}$
10^{-3}	0.5	$6.8\,x10^{-73}$
0	0.5	$5.8\,x10^{-79}$

Fusion rate evaluated as function of ρ (point where the Morse potential is linked by Coulomb, and r (force nuclear radius). The distance are reported in units of the Bohr radius.

The Screening Effect

The electrons in a metal should become a Fermi gas and the hydrogen nuclei interacting via screened coulomb potential. The effective potential between two nuclei *V(r)* which includes the effects of electron screening is given, in a simple Thomas-Fermi model, by:

$$V(r) = \frac{e^2}{r}\exp\left[-\frac{r}{\lambda}\right]$$

of course λ is the screening length and depends on density. But for $r \ll \lambda$ we can write at first order:

$$V(r) = \frac{e^2}{r} - V_0$$

This constant V_0 would be just the difference between electronic energy of a *He* isolated atom (*-79.0 eV*) and the binding energy of two H atoms (*-51.8 eV*).

The fusion rate has been evaluated using:

$$A = \nu P_n$$

Where ν is the vibrational frequency of the crystal's zero point motion (hν is about 1 eV) and P_n is the probability of a d-d nuclear reaction once the nuclei have made it to r_n.

In other words the fusion rate is calculated by multiplying *P* by the frequency of attacks on the coulomb barrier and the probability of a nuclear reaction.

The classical plasma frequency will be:

$$\omega_d = \frac{e}{\sqrt{m}}\sqrt{\frac{n_d N}{V}}$$

so the geometrical contribution is

$$\sqrt{\frac{6}{\pi}} = 1.38$$

and finally we can compute

$$\omega_d = 41.5\ eV/\hbar$$

These charge oscillations produce a screening potential having an harmonic features:

$$eV(r) = -Z_d \frac{ke^2}{2a_0} r^2$$

within a palladium lattice the Coulomb potential between two deuterons has the following expression:

$$V(r) = \frac{e^2}{r} - 85eV$$

In this case it is very easy evaluate that the intermolecular distance between two deuterons can reach the value of *0.165 Å*. Finally by means of equations and using $A=10^{22}\ sec^{-1}$, it is computed $\Lambda=10^{-22}\ sec^{-1}$.

An Effective Potential Proposed

To fit a such 'Coulomb-Morse linked' potential we have proposed the following effective potential:

$$V(r) = k\frac{e^2}{r}\left(V_M(r) - \frac{A}{r}\right)$$

where

$$V_M(r) = D'\left[e^{-2\gamma(r-r_0')} - 2e^{-\gamma(r-r_0')}\right]$$

Here A, D', γ and r_0' are parameters to determinate by means of fitting.

In the ionized hydrogen molecular the equilibrium distance is about *1.06 Å*, but in the neutral hydrogen molecular it is about 0.7 Å. We can interpret this results saying that the screening potential due to second electron whose magnitude is:

$$2 * \frac{26{,}9}{a_0} = 53.8eV$$

(here a_0 is the Bohr radius), reduces of about 34% the equilibrium distance.

Table 2	
ρ = 0.165 Å	A=0.0001
r_0 = 0.35 Å	r'_0=0.99
D = 9.34 eV	D'=0.28
	γ=1.04

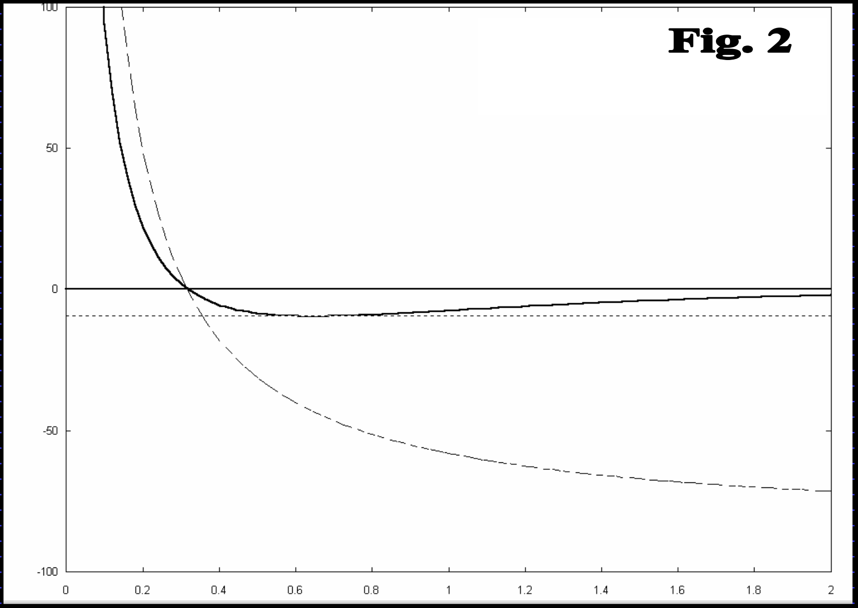
In the left column are shown the physical quantities that must characterize the potential

$$V(r) = k\frac{e^2}{r}\left(V_M(r) - \frac{A}{r}\right)$$

in the right the model parameter values that need used in the same expression in order to obtain the physical values reported in the left column.

In table 2 were reported the ρ, r_0 and D evaluations supposed and the parameters values of potential $V(r) = k\frac{e^2}{r}\left(V_M(r) - \frac{A}{r}\right)$ able to reproduce these quantities,

while in the fig. 2 is shown the feature of the same potential obtained using the values of table 2. Note the good agreement with the coulomb potential for $r < \rho$.

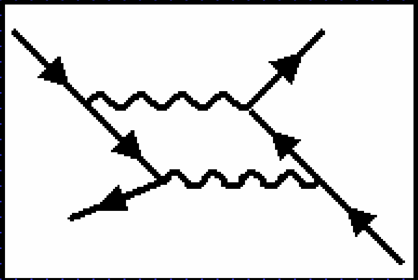


- The solid line is shows the features of potential $V(r) = k\frac{e^2}{r}\left(V_M(r) - \frac{A}{r}\right)$ *computed using the values reported in table 2.*

- The dashed line the coulomb potential $V(r) = \frac{e^2}{r} - 85eV$

Note they cross the x-axes in the same point. In the x-axes is reported the distance in Bohr radius unit and on the y-axes the energy in eV.

This attractive force is due to exchange of plasmons between two deuteron-lines as reported in figure 3:



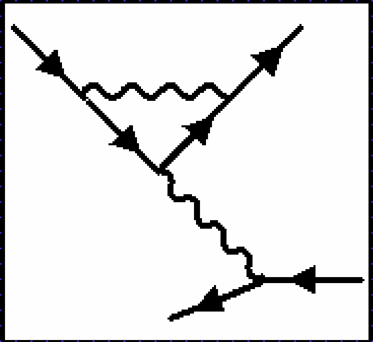


Fig 3. Plasmon exchanges. Solid lines indicate deuterons and wiggly lines plasmons.

Table 3	
ρ = 0.165 Å	A=0.0001
r_0 = 0.35 Å	r'_0=0.99
D = 50 eV	D'=1.49
	γ=1.04

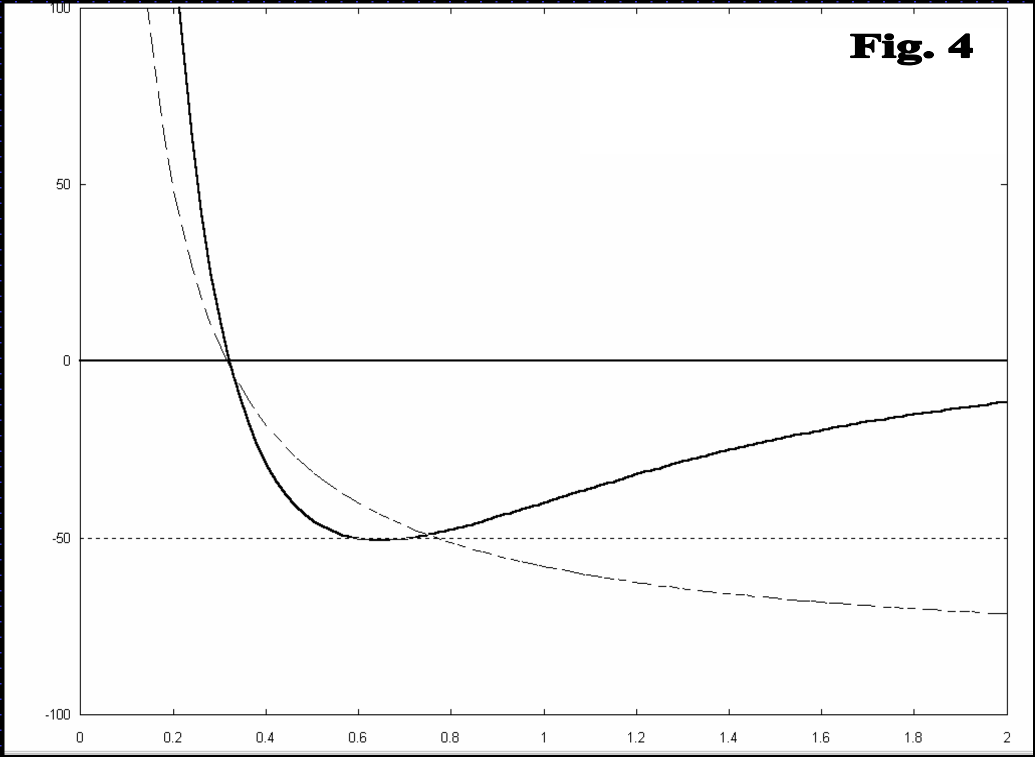
In the left column are shown the physical quantities that must characterize the potential

$$V(r) = k\frac{e^2}{r}\left(V_M(r) - \frac{A}{r}\right)$$

where D has been evaluated in the volume of M. Baldo and R. Pucci; in the right the model parameter values that need used in the same expression in order to obtain the physical values reported in the left column.

For this reason we believe that only the *D* value is reasonable.
The table 3 shows the new set of parameterization values correspondently to the alternative *D* evaluation, while the figure 4 shows the potential evaluated assuming *D = 50 eV*.

$$V(r) = k\frac{e^2}{r}\left(V_M(r) - \frac{A}{r}\right)$$



Taking into account the value reported in the previous figure we can estimate this rough dependence:

$$r'_0 = 0.35 * (50/D')$$

that can be rewritten as:

$$A = HR$$

where *R* is about nuclear radius (*20 F = 3.76* 10^{-4} a_0).
It means that *H= 0.265 eV*.

Conclusion

From data shown in table 4 we can see that the fusion rate appears in any case enough ‘great’ or, in other words, ‘measurable’. Moreover these values are in agreement with experimental data reported by S. Aiello, K. E. Rehm and Š. Miljianič.

=1.49 0.0001

TABLE 4			
A=0.0001 r_0'=0.99 D'=0.28 γ=1.04 T(K) = Const E(eV)		A=0.0001 r_0'=0.99 D'=1.49 γ=1.04 T(K) = Const E(eV)	
E ≈ 50	R ≈ $6.32 * 10^{-18}$	E ≈ 50	R ≈ $3.02 * 10^{-16}$
E ≈ 75	R ≈ $9.71 * 10^{-18}$	E ≈ 75	R ≈ $9.12 * 10^{-17}$
E ≈ 100	R ≈ $9.95 * 10^{-19}$	E ≈ 100	R ≈ $8.2 * 10^{-17}$
E ≈ 125	R ≈ $1.05 * 10^{-20}$	E ≈ 125	R ≈ $1.1 * 10^{-18}$
E ≈ 150	R ≈ $5.6 * 10^{-21}$	E ≈ 150	R ≈ $9.15 * 10^{-19}$

Fusion rate evaluated using the effective potential for different values of energy and for two different set of model parameters.

Labeled by *J* the impurities concentration, we have: $D' \rightarrow D' + GJ$

where *G* is a constant that depends on dopant-metal system.

The other parameter r'_0 will change according to the $A = HR$

while regarding A we suppose that in the presence of impurities we'll can write: $A = J\xi KT$

where ξ is a constant that depends on dopant-metal system.

- From a point of engineering view we can compute the new values of formulas

$D' \rightarrow D' + GJ$ and $A = J\xi KT$

measuring the bulk plasmon excitations in function of impurities density, and then following a set of theoretical calculations.

- Finally using the potential $V(r) = k\frac{e^2}{r}\left(V_M(r) - \frac{A}{r}\right)$ we will able to evaluate the new fusion rate.

The last point that will be evaluated in an other work is the role of micro-crack.

In fact if within a lattice a micro-crack happens a local electrical field takes place that is able to increase the deuteron energy and then according to the values of table 4 is able to enhance the fusion probability.

To conclude, we can say that a cold fusion phenomenon is real !

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