

Self-organized ignition of a tokamak fusion plasma

K. Schoepf

Institute for Theoretical Physics, University of Innsbruck, Austria

Email: klaus.schoepf@uibk.ac.at

Abstract - A new criterion for fusion ignition in magnetically confined plasmas is instituted on the basis of a plasma state analysis determined by the non-stationary particle and energy reaction-thermal interaction. The application of a soft Troyon β -limit, together with the actual fusion power deposition and its effect of deteriorating the plasma energy confinement are crucial parts of the fusion burn dynamics which determine the ignition conditions. Considering continuous fuelling we find that the separatrix established by the dynamic trajectorial evolution in the state space is the critical boundary which must be exceeded in order that the plasma evolves by itself towards ignition without any auxiliary heating. Thus no longer the simple zero-power contour commonly referred to is the critical boundary but rather, in the dynamic sense, the appearance of the plasma state in a basin of attraction by the stable equilibrium point. Further, the minimum heating path toward stable ignition is examined.

Introduction

The continuous progress in the attainment of plasma parameters required for establishing nuclear fusion in magnetically confined plasmas as well as the prospect of feasible steady-state operation has instigated the interest in the physics of burning plasmas [1]. Aside from the required plasma current drive, fusion energy production with tokamaks demands particular attention to confinement and fuelling regimes in order to maintain the plasma density n and temperature T at favourable values matching with specific requirements such as the triple product $n\tau_E T$, where τ_E represents the plasma energy confinement time. The identification of state and parameter space regions capable of ignited fusion plasma operation is evidently crucial if significant energy gains are to be realized over longer periods.

In this study the particle and energy dynamics of a d-t fusion plasma are considered as effected by heating schemes, by fuelling, by fusion power deposition, and by transport as well as radiation power losses in an ITER-like tokamak capable of ignition [2]. After identifying the ignition contours, the region of ignited operation and the fixed points in the relevant state space, we will examine the typical nonlinear features of a burning fusion plasma evolving towards or, respectively, repelled from equilibrium points of the dynamic [3]. Interestingly, some plasma states outside the ignition contour will be seen to be attracted by a stable fixed

point on the ignition contour [4] thus rendering possible a self-organized ignition driven solely by constant fuelling.

Burn Dynamic Model

We start from an axisymmetric description incorporating the radial profiles of the densities and temperatures of the plasma constituents in an elongated poloidal cross section featuring triangularity. The space and energy dependent fusion alpha particle distribution is found from of a reduced slowing down kinetic equation taking into account TF ripple losses and TAE diffusion. Assuming charge neutrality the alpha particle population and the fuel ion density determine the electron density. Allowing for different electron and ion temperatures reveals the respective effects on them by different heating mechanisms. Because energy confinement degradation due to alpha particle heating is to be quantitatively proved yet and hence its consideration in confinement time scalings bears a somewhat arbitrary degree of uncertainty, we calculate here the energy confinement times for each species via theoretically derived heat conductivities [5] and also adopt empirical scaling laws [6,7]. Integrating the basic kinetic equation over the entire velocity space and taking different relevant parabolic radial profiles of ion and electron kinetic temperatures, T_i and T_e [keV], as well as of fuel ion and electron densities, n_i and n_e , we perform volume averaging of the local particle and power balance equations for each plasma species and arrive at a descriptive set of coupled nonlinear differential equations determining the dynamic evolution of the fusion plasma state:

$$\frac{d\bar{n}_i}{dt} = \bar{s}(t) - \frac{\bar{n}_i}{\tau_i} - \frac{1}{2} \overline{n_i^2(r, \chi, t) \langle \sigma v \rangle_{dt}(T(r, \chi, t))} \quad (1.1)$$

$$\begin{aligned} \frac{d\left(\frac{3}{2} \overline{(n_i T_i + n_e T_e)}\right)}{dt} = & \overline{P_{aux}(r, \chi, t)} + \overline{P_{Ohm}(r, \chi, t)} + \eta_\alpha \overline{P_\alpha(r, \chi, t)} \\ & - \frac{3}{2} \frac{\overline{(n_i T_i + n_e T_e)}}{\tau_E} - \overline{P_{brems}(r, \chi, t)} - \overline{P_{cycl}(r, \chi, t)} - \overline{P_{line}(r, \chi, t)} \end{aligned} \quad (1.2)$$

Here is \bar{s} the volume averaged fuelling rate, τ_p the average particle confinement time which is taken to be five times the global energy confinement time τ_E , further $\langle \sigma v \rangle_{dt}$ denotes the d-t fusion reaction rate parameter averaged over the distribution functions of reactants, while r and χ represent the radial and poloidal coordinates; the several power density terms in Eq.

(1.2) are self-explanatory and η_α designates a fusion energy transfer factor calculated from a kinetic equation including TF ripple transport and TAE diffusion of alphas [8]. Hence η_α accounts for the fraction of the average d-t fusion power density that is retained in the plasma. If auxiliary heating, P_{aux} , is provided by neutral beam injection (NBI), non-thermal fusion reactions involving fast ions generated by NBI are accounted for as well. Assuming a constant alpha impurity density of $0.03 \bar{n}_i$ and quantifying the average electron density as determined by charge conservation, and further expressing the various power density terms as explicit functions of $\bar{n}_i, \bar{n}_e(\bar{n}_i), \bar{T}_i, \bar{T}_e$ as in Refs. [3,8,9], the resulting dynamic variables in this extended globally averaged formulation of Eq.(1) are the average ion density \bar{n}_i , the average ion temperature \bar{T}_i and the average electron temperature \bar{T}_e . Their temporal changes form the dynamic system

$$\begin{aligned} \frac{d\bar{n}_i}{dt} &= f(\bar{n}_i, \bar{T}_i, \bar{T}_e, \bar{s}) \\ \frac{d\bar{T}_i}{dt} &= g(\bar{n}_i, \bar{T}_i, \bar{T}_e, \overline{P_{aux \rightarrow i}}) \\ \frac{d\bar{T}_e}{dt} &= h(\bar{n}_i, \bar{T}_i, \bar{T}_e, \overline{P_{aux \rightarrow e}}) \end{aligned} \quad (2)$$

where the specific functions f, g, h are specified by the functional dependencies of the terms occurring in Eq.(1) and by their averaging. Note that the global energy confinement time τ_E used here was expanded as function of plasma and device parameters such that it comprehends all relevant operation regimes, i.e. Ohmic heating, L-mode, transition to H-mode depending on a threshold power density, and the H-mode regime [8]. In addition, to model the confinement degradation as the Troyon beta limit β_T [10] is approached – experimental observations indicate a 20% increase of transport losses at $0.8\beta_T$ –, a soft beta limit (*SBL*) is introduced here by multiplying τ_E with the factor

$$SBL = 1 + \frac{35(\beta / \beta_T)^2}{\pi \left[1 + 25(1 - \beta / \beta_T)^2 (3\beta_T / C_T)^2 \right]}, \quad (3)$$

where β is the usual ratio of kinetic plasma pressure to magnetic pressure and

$$\beta_T [\%] = C_T \frac{I_p [\text{MA}]}{a [\text{m}] B [\text{Tesla}]} [\%] \quad \text{with the Troyon factor } C_T \text{ and the total plasma current } I_p. \text{ This}$$

SBL effectively eliminates all of state space above β_T as a possible operating regime and is evidently a more plausible scenario than modelling which only begins to appreciably degrade confinement at and beyond the Troyon limit.

Three-Dimensional Dynamic

The 3 state variables $\bar{n}_i, \bar{T}_i, \bar{T}_e$ define the 3-dimensional plasma state space, in which the fusion dynamic is illustrated. To inspect not only the dynamic behaviour in the neighbourhood of the fixed points, numerical integration (Runge-Kutta method of order 8) of the initial value problem was performed for various initial conditions. All these different initial situations provide a variety of trajectories elucidating the respective plasma state evolution. For a clearer demonstration of the trajectorial tracks, the ones starting at phase space points with $\bar{T}_i = \bar{T}_e$ are indicated in Fig. 1 by full lines, while the dashed lines refer to those with initial positions featuring $\bar{T}_i < \bar{T}_e$.

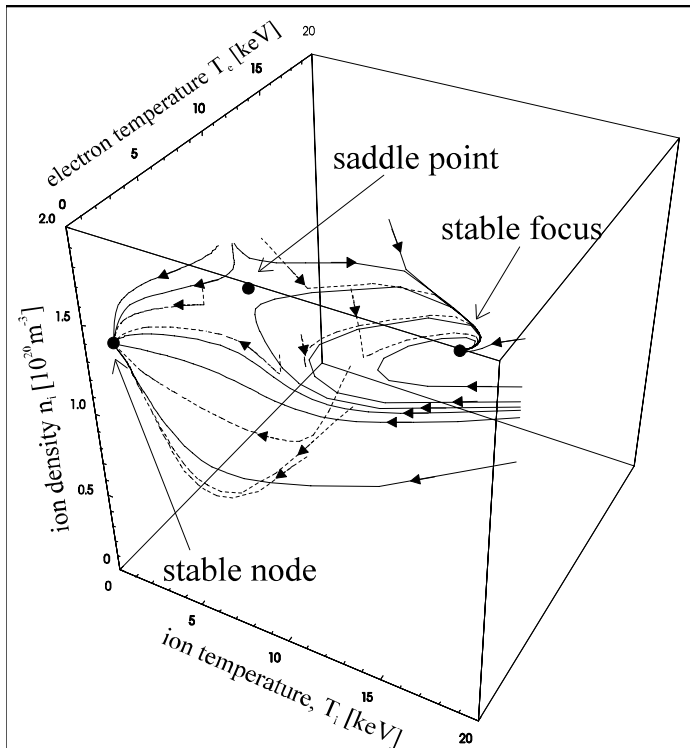


Fig.1: *State space illustrating the dynamic behaviour of the system. Depending on the initial conditions the system will evolve along the displayed trajectories if no additional heating mechanism is applied ($C_T=3.5$, $\overline{P_{aux}}=0$, $\bar{s}=0.1 \times 10^{20} m^{-3} s^{-1}$, no impurities except alpha particles).*

Characterized by the attracted trajectories, two stable fixed points become evident in Fig. 1. The one at low plasma temperature represents just the so-called Ohmic ignition, that is when Ohmic heating balances the plasma power loss. Since the fusion reactions contribution at such low temperature is negligible, this equilibrium point is far from the fusion ignition

range. However, both the stable focus as well as the repelling unstable saddle point lie on the fusion ignition contour defined by the stationary energy balance

$$\frac{dE_p}{dt} = \frac{d}{dt} \left[\frac{3}{2} \left((n_i + n_{\alpha,th}) T_i + n_e T_e \right) \right] = 0 \text{ for the case } \overline{P_{aux}} = 0, \quad (4)$$

where the thermal plasma energy E_p incorporates the thermalized alphas as well. From this definition of the ignition contour a surface in the 3-dim state space is expected as solution. In

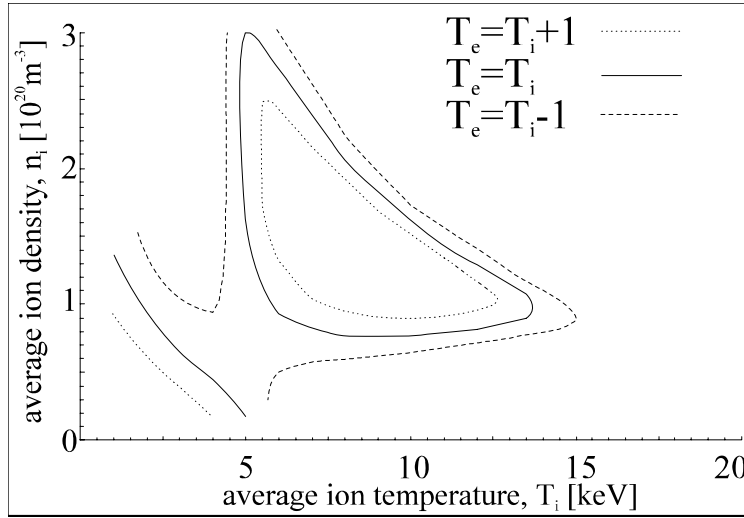


Fig.2: Ignition contours in the (n_i, T_i) -state plane for different electron temperatures ($C_T=3.5$, $P_{aux}=0$, $\bar{s} = 0.1 \times 10^{20} m^{-3} s^{-1}$; no impurities except alpha particles)

Fig. 2 some ignition contours are displayed in a two dimensional $\bar{n}_i - \bar{T}_i$ state plane for different electron temperatures and give an impression of the full ignition surface in the $\bar{n}_i - \bar{T}_i - \bar{T}_e$ space. For $\bar{T}_e \geq \bar{T}_i$ the ignition contour at fixed \bar{T}_e appears to feature two separated parts, Ohmic and fusion ignition, with the actual ignition region significantly decreasing as \bar{T}_e is raised, which occurs due to enhanced radiation losses, until it will vanish if $\bar{T}_e - \bar{T}_i$ exceeds a critical value. On the other hand, as seen in Fig. 2, the ignition region becomes larger in the case of $\bar{T}_e < \bar{T}_i$ and, for some critical small difference between electron and ion temperature, the Ohmic ignition contour even runs into the other contour forming then only a single ignition curve. The enlargement of the ignition contour can be explained in this case by lower radiation losses and due to a larger energy transfer function η_α which increases with decreasing plasma temperature.

From inspection of the course of trajectories in Fig. 1 the existence of a surface can be conceived which separates the respective basins of attraction of the two stable fixed points. A

trajectory starting near this separatrix face will be repelled. On the other hand, the trajectories originating in the saddle point and proceeding to the two stable fixed points draw up nearby trajectories and thus constitute an attracting separatrix. The depiction of these separatrices is unmanageable in a 3-dimensional plot and hence they are not seen in Fig.1. Trajectories starting from a state with $\bar{T}_i \neq \bar{T}_e$ are observed to be quickly drawn towards the $\bar{T}_i = \bar{T}_e$ - plane due to the small electron-ion energy exchange time τ_{ei} at the typical values of plasma temperatures and densities of interest. Since $\tau_{ei} \ll \tau_E < \tau_p$, the dynamic variations are, in most cases, insignificant during temperature equilibration and the trajectorial investigation for $t \gg \tau_{ei}$ may be reduced to a more intelligible 2-dimensional dynamic in the $n_i - T$ state plane with $T = \frac{\bar{T}_i + \bar{T}_e}{2}$ being the average plasma temperature and n_i the average ion density, wherein the bar is suppressed from now on. Only when the initial states exhibits a sizable difference between \bar{T}_i and \bar{T}_e , as well as when auxiliary heating is specifically delivered to plasma ions (ICRH) or electrons (ECRH) and thus sustains $\bar{T}_i \neq \bar{T}_e$, a 3-dimensional dynamic needs to be investigated as in Fig. 1.

Two-Dimensional Dynamic

Analyses distinguishing ion and electron temperature revealed only an insignificant difference (< 1 keV) between the two on the time scale (t) of the burn dynamic unless there was substantial preferential auxiliary heating of one species. Thus, a single temperature formulation is deemed sufficient here for a perceptive demonstration of the ignition dynamics, which then are determined by the evolution equations

$$\frac{dn_i}{dt} = f(n_i, T, \bar{s}) \quad (4.1)$$

$$\frac{dT}{dt} = g(n_i, T, \overline{P_{aux}}) . \quad (4.2)$$

Solving these equations yields the trajectories and separatrices of Fig. 3, where also the ignition contour is indicated embracing a state region with positive power balance $dE_p / dt > 0$. Further shown is the plasma state limitation by β_T . Two stationary points of the system dynamic (solutions from equating Eq. (4.1) and Eq. (4.2) to 0 with $P_{aux} = 0$) are evident, one an attractor and the other a saddle point. The Ohmic ignition point is not

displayed as it is of no interest due to the inferior fusion power there. Of greater importance, however, is the dynamic evolution of the system, i.e. the non-steady state solutions $n_i(t)$ and $T(t)$ yielding the trajectories and separatrices in Fig. 3. Apparently, all states above separatrix 2 will evolve toward the ignited stable node, if only constant fuelling and no

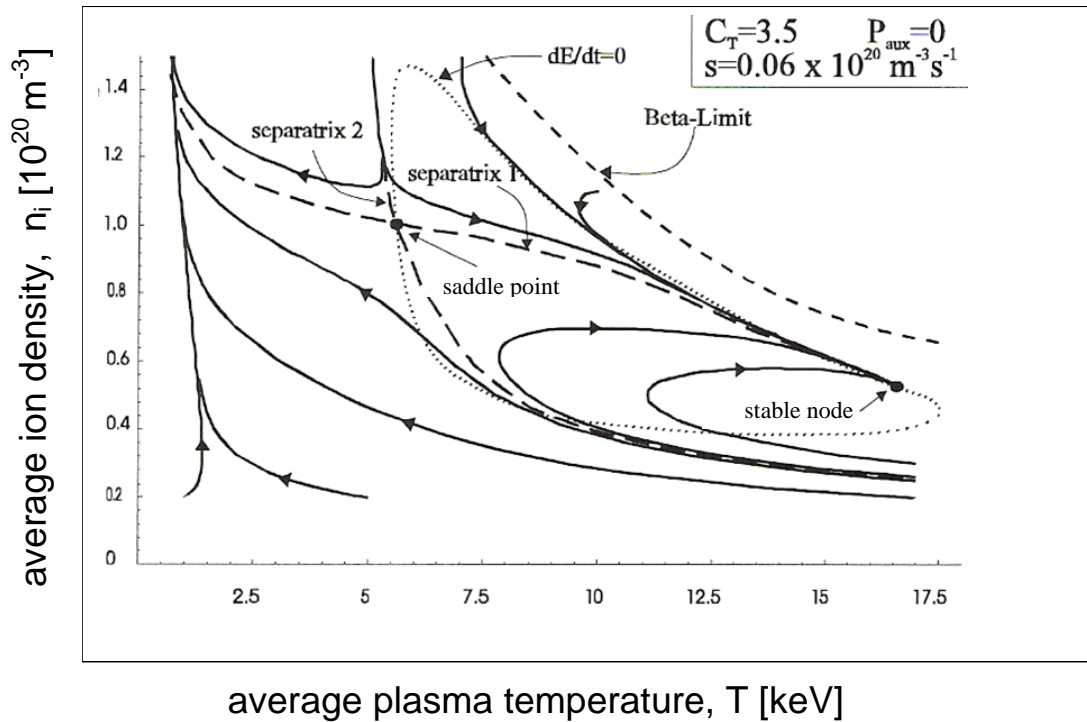


Fig. 3: Global $d-t$ fusion dynamics in the density-temperature state plane. The zero power curve, $dE_p/dt=0$, identifies the ignition contour. Depending upon the initial conditions, the system will evolve along the trajectories shown if not any control mechanism or auxiliary heating is established.

auxiliary heating or controlling is applied. Interestingly, a high temperature, low density region from which the stable ignited equilibrium point is attained dynamically only by fuelling exists outside the zero-power ignition contour. Plasma states in this region will evolve – without any auxiliary heating or power control – to, and remain at, the stable high temperature attractor. This process of ‘self-organized ignition’ is not foreseeable solely through steady state power balance ignition studies and thus alters the regimes of the n_i-T state plane which are to be called ignited. So the ignition criterion for an ITER-like fusion plasma is not merely achieving a plasma state within the stationary zero-power contour, but rather the separatrix 2 of Fig. 3 must be exceeded for the system to evolve on its own towards conventional fusion ignition. Conversely, plasma states in the region below separatrix 2 are not fusion-ignited since – without auxiliary heating – they evolve to the low temperature, high

density, Ohmic ignition regime associated with negligible fusion power. The location and nature of the ignited attractor as well as the form of the essential separatrix have been shown to vary strongly with the fuel injection rate [3]. To illustrate this effect we display in Fig. 4 the fusion ignition dynamic of a tokamak d-t plasma continuously fuelled at a rate density substantially smaller than that in the case of Fig. 3. Trajectories in the basin of attraction above the separatrix in Fig. 4 are seen to spiral into the ignited operation point that is classified now as a stable focus.

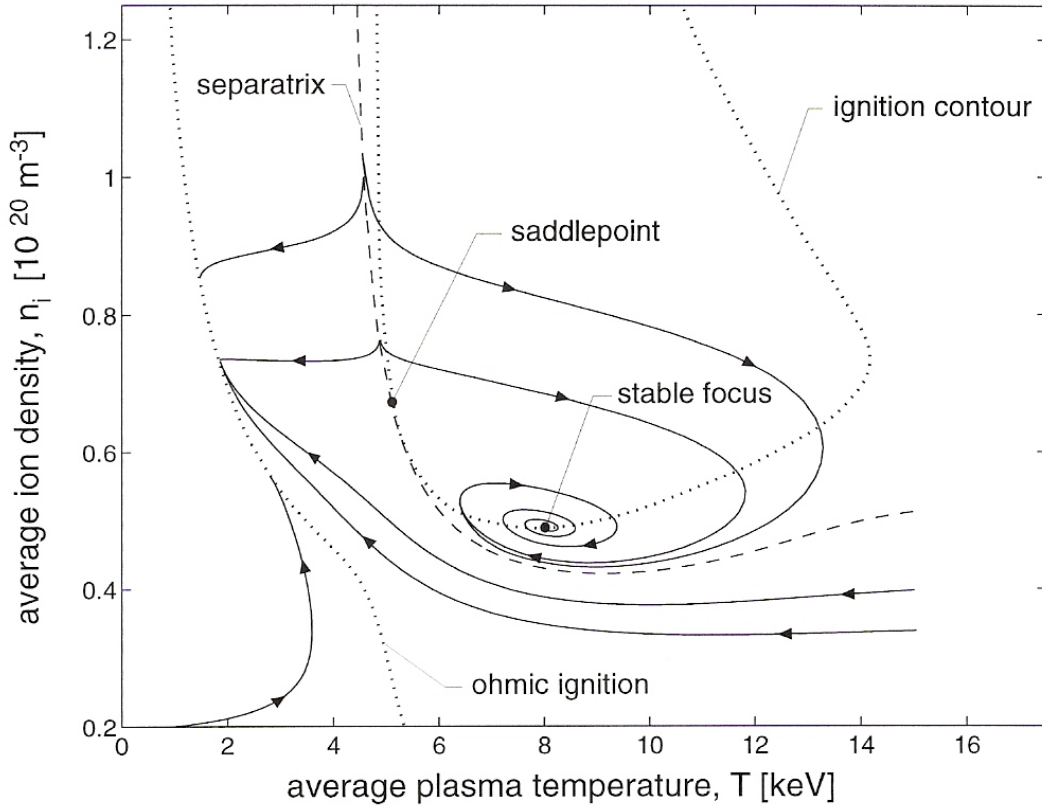


Fig. 4.: Trajectory dynamic of an ITER-like fusion plasma ($C_T=3.5$, $P_{aux}=0$) in the density-temperature state plane for a constant fuelling rate $\bar{s} = 0.014 \times 10^{20} \text{ m}^{-3} \text{ s}^{-1}$ resulting in stable focus as the high temperature fixed point.

By further reducing the fuelling rate it appears even possible to bend the right hand tail of the separatrix such that it ends in the saddlepoint thus enclosing the stable focus. Such a fuelling regime ($\bar{s} = 1.3 \times 10^{18} \text{ m}^{-3} \text{ s}^{-1}$) renders the transgression of the separatrix and subsequent self-ignition at minimum requirements for plasma density and temperature. The corresponding case of minimum auxiliary heating till the onset of self-organized dynamic ignition is displayed in Fig. 5. After a sole Ohmic heating phase over 20 s the plasma is heated by neutral beam injection (NBI) delivering 226 MJ into the core. This brings the plasma into a state slightly beyond the separatrix where the auxiliary heating mechanism can

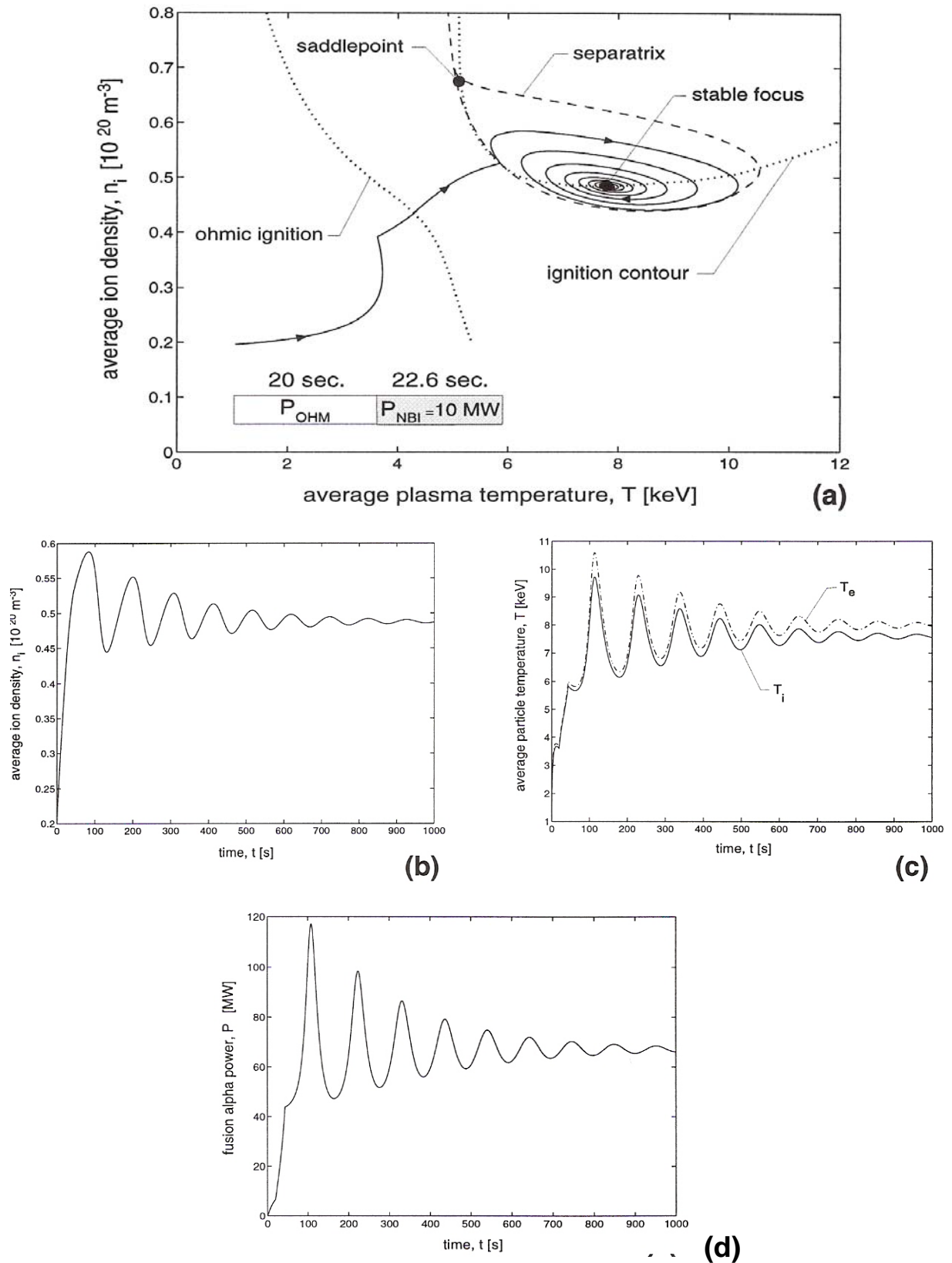


Fig. 5: (a) Ignition path associated with minimum auxiliary heating energy. The dt plasma is considered to be fuelled by a constant average rate density of $\bar{s} = 0.013 \times 10^{20} \text{ m}^{-3} \text{ s}^{-1}$. Temporal evolutions along this ignition path of the plasma ion density, of the ion and electron temperature as well as of the fusion alpha power are displayed in figures (b)-(d).

be turned off and the process of self-ignition begins. On its way to stable ignition the evolution trajectory may – if the fixed point is a focus – spiral around that and hereby leave and re-enter the ignited region consecutively. For NBI we assumed here the injection of 500 keV deuterium atoms and hence beam-target fusion reactions had to be considered as well. We note that fuelling the plasma at a rate lower than $1.3 \times 10^{18} \text{ m}^{-3}\text{s}^{-1}$ only allows for operation regimes without a stable ignited fixed point. The heating route presented in Fig. 5, though providing a clear idea of the minimum energy path, may not be desirable for practical reactor operation because the dynamic approach of the stable focus is seen to go along with relatively large density and temperature alternations over a long time period ($\Delta T = \pm 1 \text{ keV}$ for $200 \text{ s} < t < 300 \text{ s}$). Therefore we propose in Fig. 6 a more applicable path towards fusion ignition avoiding such fluctuations of n_i , T_i and T_e . Following an initial sole Ohmic heating phase the energy from 9 MW ion cyclotron resonance wave heating (ICRH) impels the plasma directly to the same stable operation point as in Fig. 5 after 42 s. Upon terminating ICRH only insignificant variations of plasma parameters are observed.

If stable operation is now preferred at higher values of n_i and T , there is no need for further auxiliary heating: we rather increase slightly the constant fueling rate such that the newly formed separatrix lies somewhat higher but still below the old operation point. Upon that the plasma will dynamically evolve by itself into an ignited state determined by the new attractor appearing at larger n_i and higher T [4]. Appropriate repetition of this procedure can drive the fusion plasma towards any stable equilibrium point without auxiliary heating.

To keep the first wall damage low, operation in a subignited region may be envisaged. Since there exists no equilibrium, one has to think of stabilizing mechanisms in order to maintain a constant power level in the reactor. Stabilization of operation points in subignited regions may be simply performed by suitably suppressing power or, respectively, fuel supply as has been shown in Ref. [8] where also a control mechanism of the unstable, ignited fixed point is suggested by means of small auxiliary heating powers.

Conclusions

Our fusion burn dynamics study has instigated a novel realization of fusion plasma ignition. We conclude that the criterion for possible ignited operation of a magnetically confined fusion plasma is the crossing of the separatrix in the n_i - T state plane or, correspondingly, the transgression of a separatrix-surface in the n_i - T_i - T_e state space – a consequence of the nonlinear dynamics of the system – as the common stationary ignition

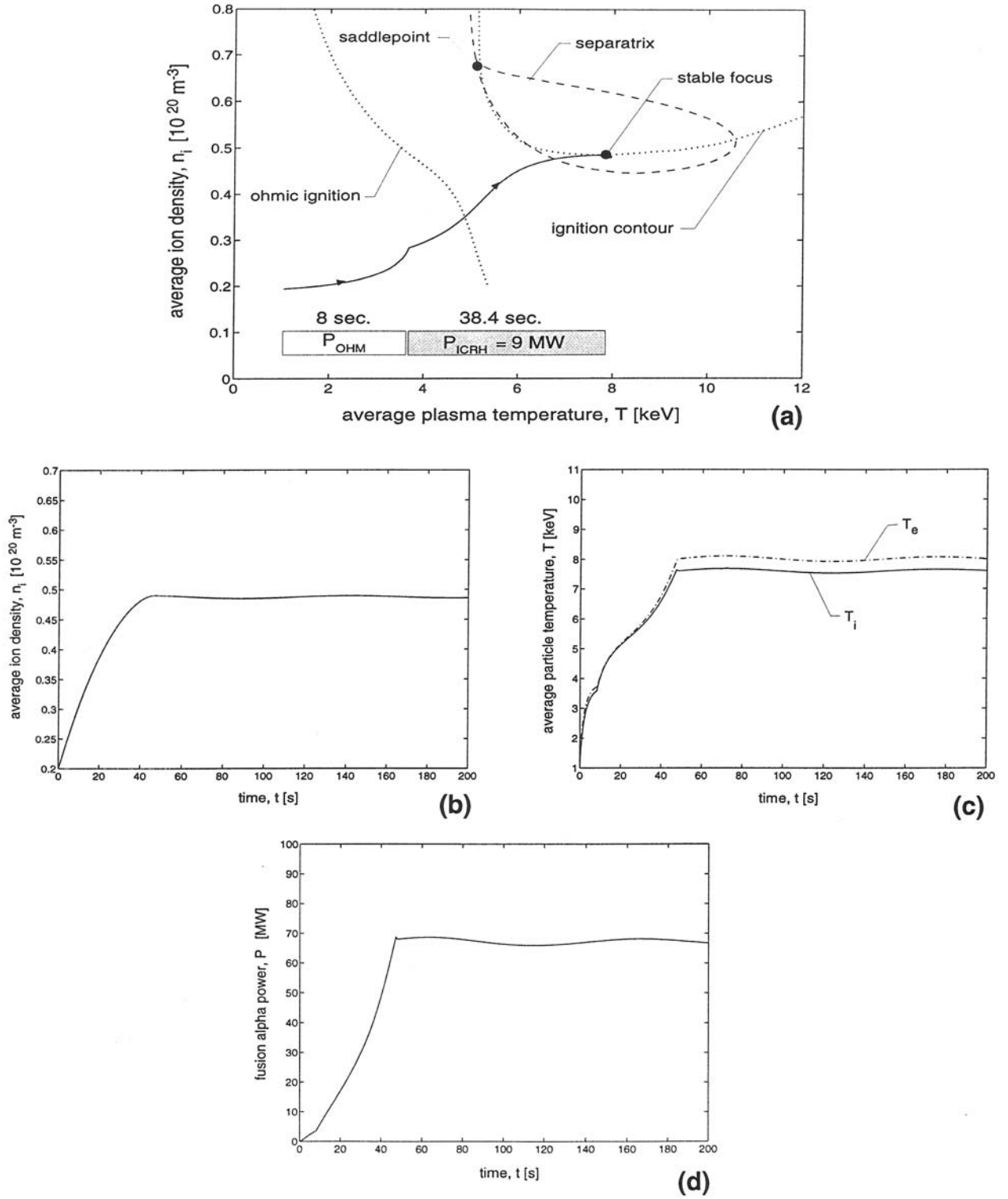


Fig. 6: (a) Preferable path to an ignited stable operation point of an ITER-like plasma with low fusion power production. Here all plasma parameters correspond to Table 1 in the Appendix and fuelling is managed at an average rate density of $\bar{s} = 0.013 \times 10^{20} \text{ m}^{-3} \text{ s}^{-1}$. Figures (b)-(d) exhibit the temporal evolution of ion density, plasma temperature and of the fusion alpha power produced.

contour or the power balance alone do not completely reveal which plasma states evolve by means of self-organization, i.e. in the absence of auxiliary heating or control power, to the stable ignited fixed point. Employing the fuelling rate as a control parameter, the attracting ignited operation point could be managed to appear at fairly moderate plasma conditions in ITER-like fusion devices. A region of 'dynamic self-ignition' outside the positive power balance state space was identified and the dynamic criterion of exceeding the separatrix must be recognized as an important consideration for both initial plasma heating and ignited operation of a d-t tokamak.

The known parameter discrepancy between stable and unstable ignited steady-state operation is found to be controllable by the fuelling rate, and thus may be reduced for a d-t fusion plasma to only a very few keV difference in the plasma temperature T . Further, the minimum heating path toward stable ignition was inspected; however, it was identified as an impracticable operational scenario due to substantial oscillations of the plasma density and temperature. Avoiding the latter, a heating regime with optimized low energy expense was proposed that leads conveniently to the stable ignition point at reasonable plasma temperatures in the 20 keV range. To minimize the energy required to heat the plasma beyond a state, from where it turns dynamically to ignited stable equilibrium by self-organization, a relatively high power neutral beam applied for a short duration is found to be more efficient than a less powerful beam over a longer time period.

Sub-ignited operation requires sensitive controlling and, of course, greater power input to the device. Ignited regimes turned out to be most responsive to variations of the fuelling rate, which can totally alter the trajectorial and dynamic characteristic; in certain parameter ranges already a small $\Delta\bar{s}$ may change a stable operation point into an unstable one. Further, we identified a minimum fuelling rate below which equilibrium operation points do not exist.

The objective of our future studies in this context is to examine the burn dynamics not on the global plasma average but rather retaining the dependence on the coordinates characterizing the poloidal plane in an axisymmetric toroidal device. Assuming appropriate fuel injection, these recent developments appear to confirm the possibility of a thermally stable energy self-sufficient tokamak fusion plasma in a parameter domain definitely accessible by the reactors succeeding ITER.

Acknowledgement

This work has been partially carried out within the project P4 of the Association EURATOM-OEAW and was partly funded by this organisation.

Appendix

Table 1: Basic parameters of the tokamak device referred to in this work (similar to earlier ITER designs)

$R=8.1$ m	major plasma radius	$\kappa=1.6-2$	plasma elongation
$a=3.0$ m	minor plasma radius	$\delta=0.3$	triangularity
$B_0=5.7$ T	toroidal magnetic field on axis	$N=16$ (24)	number of toroidal field coils
$I_p=18-24$ MA	total plasma current	$R_f=0.9$	first wall reflectivity
$q(a) = 3$	safety factor at the edge:	$Z_{eff}=1.6$	
$C_T = 3\div 4$	Troyon factor	$A_i=2.5$ amu	. average fuel ion mass:

References

- [1] IAEA Workshop on Burning Plasma Physics and Simulation, Tarragona, Spain, July 4-5, 2005; Workshop Summary in Fusion Sci. Techn. **49**, 79 (2006)
- [2] see Appendix, Table 1
- [3] K. Schoepf et al., KERNTECHNIK **60**, 179 (1995)
- [4] K. Schoepf, T. Hladschik, Ann. Nucl. Energy **23**, 59 (1996)
- [5] D. Sigmar, C.T. Hsu, KERNTECHNIK **54**, 51 (1989)
- [6] ITER Final Design Report Technical Basis: Plasma Performance Assessment, <http://www.iter.org> (2007)
- [7] D. Anderson et al., Fus. Technology **23**, 5 (1993)
- [8] T. Hladschik, PHD thesis, University of Innsbruck, Austria, June 1994
- [9] T. Hladschik, K. Schoepf, Proc. 1994 IWPP Current Topics in Astrophysical and Fusion Plasma Research, pp 64-69, dbv-Verlag Graz, Austria (1994)
- [10] F. Troyon et al., Plasma Phys. Contr. Fusion **26**, 209 (1984)